Creating Complex Equations and Their Graphs

by
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Crystal Clear Mathematics (2013)

Introductory Notes

Using Six Basic Equations and Zeros to Create Complex Equations and Their Graphs

Thank you for downloading this file. In it, you will find a brief summary of the equation types that we are going to use. You will also find two sets of six questions.

In order to understand the principles and skills involved in this exercise, please watch the YouTube video Using Zeros to Create Complex Graphs.

When you have completed your work, compare your answers with those provided on the last pages of this document.

Thank you and best wishes!

Graeme Henderson (Crystal Clear Mathematics)

Six Basic Equation Types

Straight Lines

Although often written in gradient-intercept form ($y = mx + b$) or standard form ($ax + by = d$), this exercise requires that you write them in general form ($ax + by +c = 0$). These equations are called 'linear equations'; their graphs are called 'straight lines' … sometimes, simply, 'lines.'

Parabolas

The term 'parabola' describes the shape of the curve, or the graph. The corresponding equation is called a 'quadratic equation.' These equations are commonly written in the form $y = ax^2 + bx + c$ but, in this exercise, we will find it useful to write them in the form $y – ax^2 – bx – c = 0$. If you prefer, $ax^2 + bx + c – y = 0$ is an equivalent statement.

Cubic Equations

The third of our polynomial equations is called a cubic equation since its highest power is 3 (i.e. it has degree three). The graph is commonly referred to as a 'cubic' and we will write its equation in the form $y – ax^3 – bx^2 – cx – d = 0$. These are typically 'S' shaped curves.

The next three graphs are not polynomial functions.

Circles

The equation for a circle, centred on the origin, is $x^2 + y^2 = r^2$ where r is the radius of the circle. We will write this equation in the form $x^2 + y^2 – r^2 = 0$.

Hyperbolae (or Hyperbolas)

The hyperbolae with which junior high school students are generally familiar are called rectangular hyperbolae. Their equations take the form $y = c/x$ or $xy = c$.

We will write them in the form $xy – c = 0$.

Exponentials

The exponential equation (or exponential function) is written in the form $y = a^x$ and its graph is also referred to as an exponential function (or simply an exponential). For this exercise we require that these formulae be written in the form $y – a^x = 0$.

A series of videos will be created on YouTube for each of these graph types. It is intended that each series will explain everything that a school student needs to understand in order to graph and analyse that particular type. For this exercise, it is assumed that you have a working knowledge of them all.

The Principle

If we string a number of our six equations together as factors in a larger equation and set the entire product equal to zero, then each individual factor may be worth zero and, therefore, the graph generated will be a composite of all the graphs described by each of the factors.

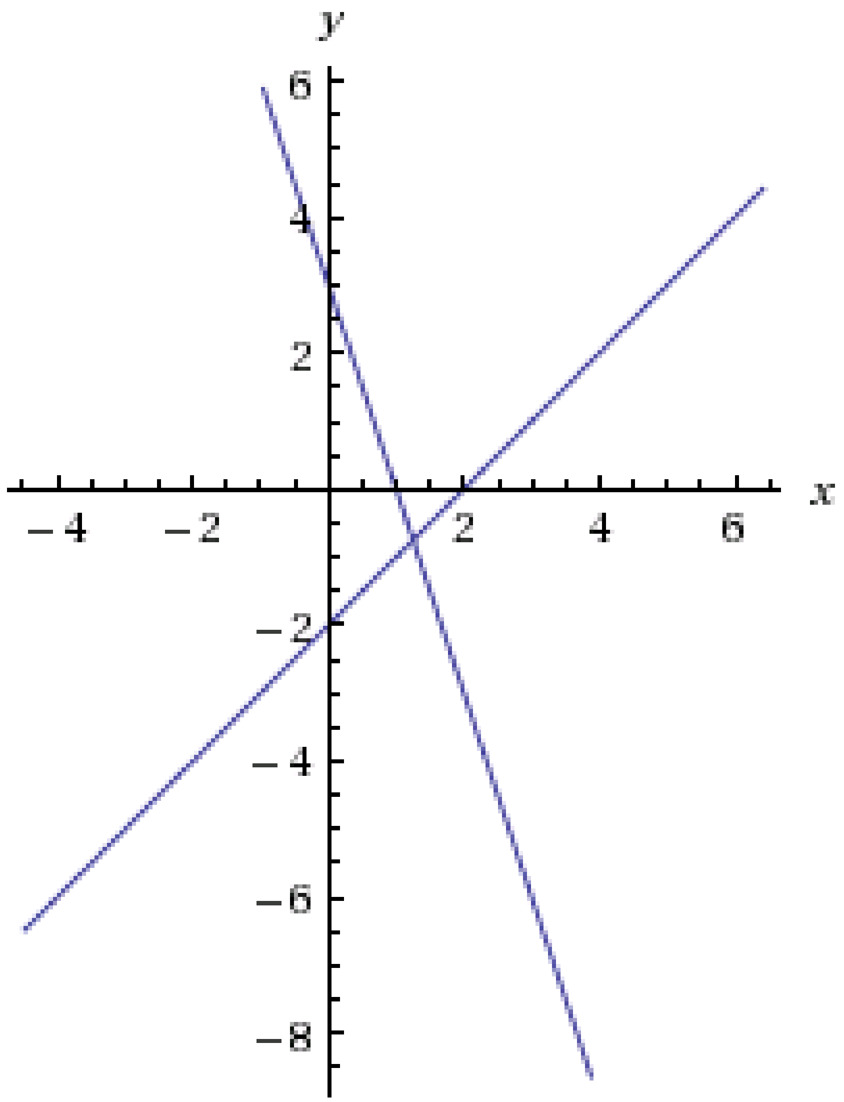
For example, the graph of $(y - x + 2)(y + 3x - 3) = 0$ is composed of two straight intersecting lines. This is because the equation is composed of two factors and, when either of them is worth zero, then the whole equation is 'satisfied.'

So, $y - x + 2 = 0$ is one solution. This is the line $y = x - 2$.

The other solution is $y + 3x - 3 = 0$. This is the line $y = -3x + 3$.

Any point on either of these two lines will therefore cause one of the two factors to be worth zero and, thereby, satisfy the composite equation!

Here is its graph!



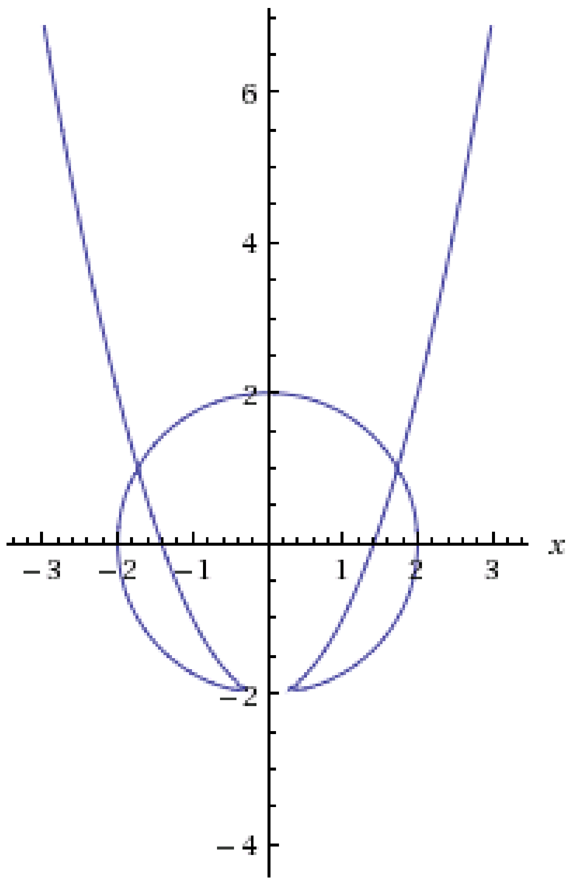
The Questions

Draw the graph for the following equations:

1. $(y - x - 1)(y + x - 1)(x^2 + y^2 - 9) = 0$
2. $(xy - 2)(xy + 2)(x^2 + y^2 - 4) = 0$
3. $(y - x^3)(y + x^3) = 0$
4. $(xy - 2)(y + x^2 + 2)(y - x^2 - 2) = 0$
5. $(y - 2x)(2y - x) = 0$
6. $(y - 3^x)(y - 3^{-x})(x^2 + y^2 - 1) = 0$

Construct equations for each of the following graphs:

1.



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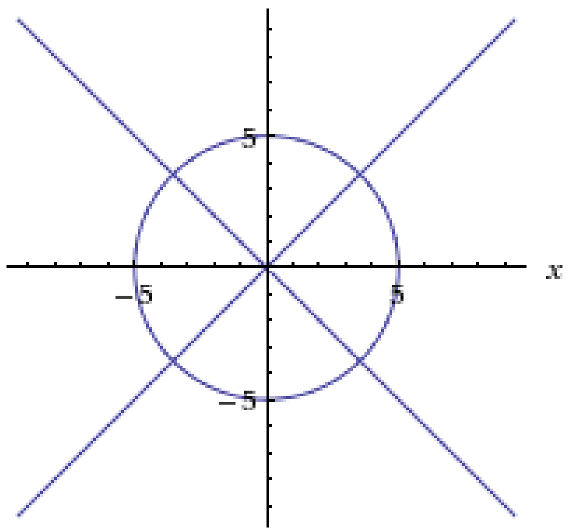

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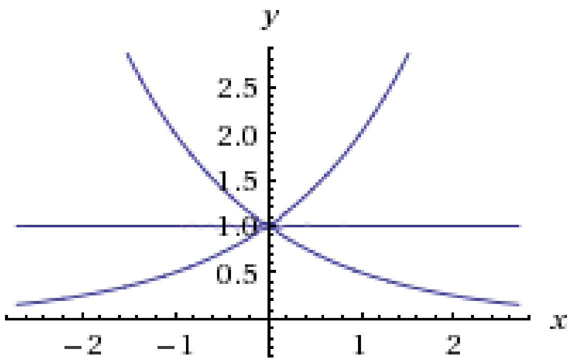
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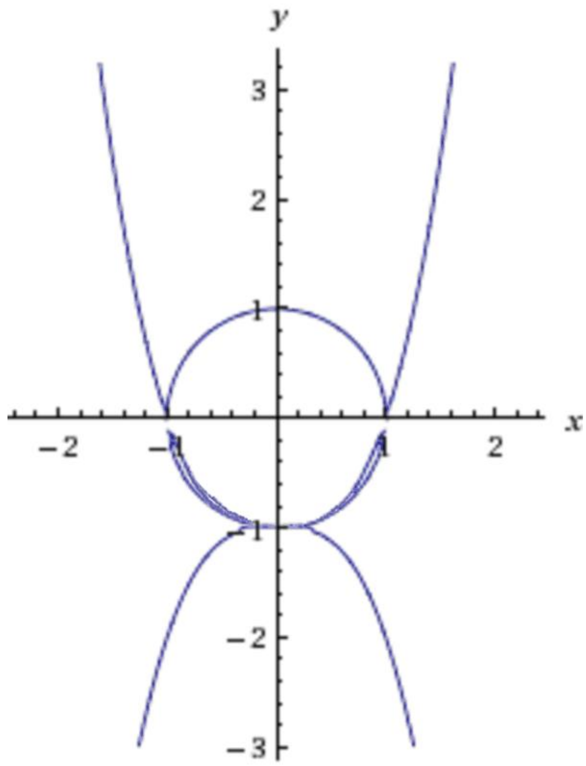
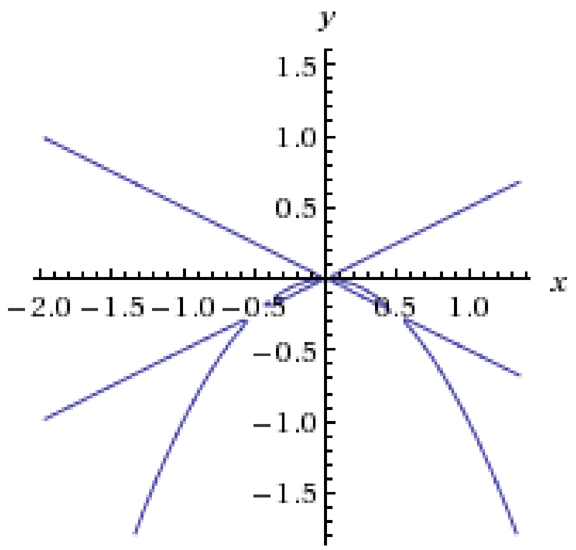
2.0
1.5
1.0
0.5
x
−2
−1
1
2

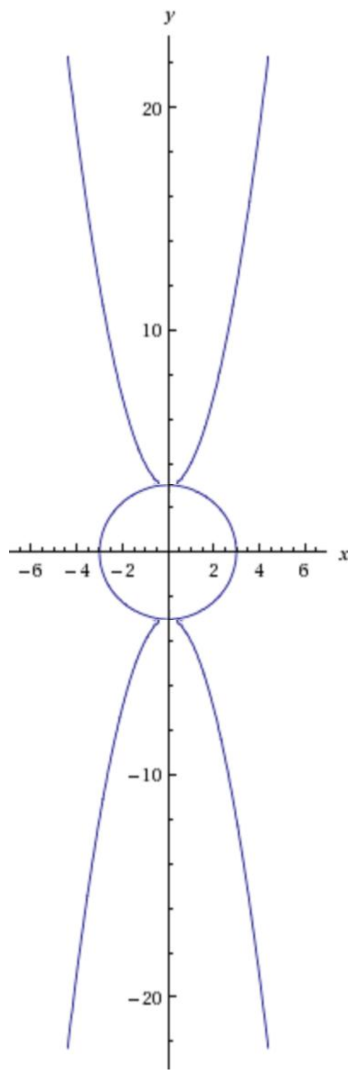
4.


y
3
2
1
x
−2
−1
1
2
−1
−2
−3

5.


y
1.5
1.0
0.5
x
−2.0 −1.5 −1.0 −0.5
0.5
1.0
−0.5
−1.0
−1.5

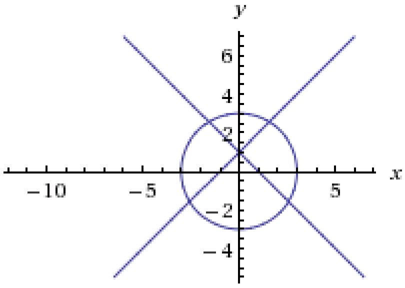
6.


y
20
10
x
−6
−4
−2
2
4
6
−10
−20

Solutions

Drawing graphs given their equations:

1.



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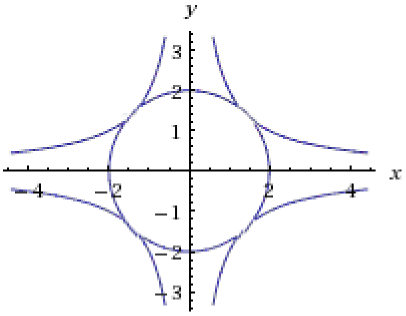

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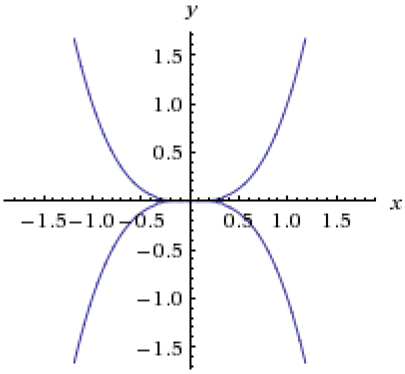

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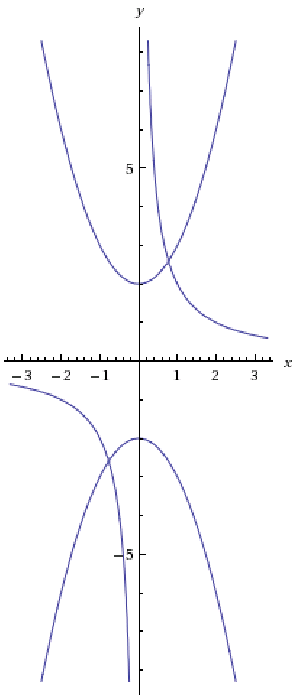

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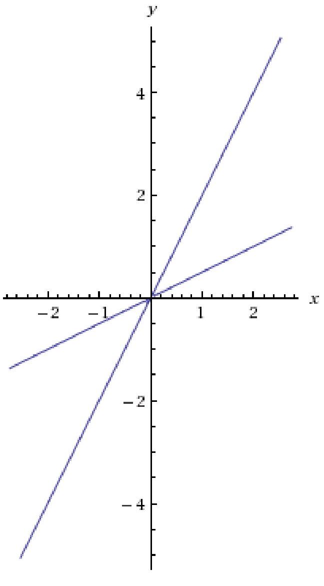

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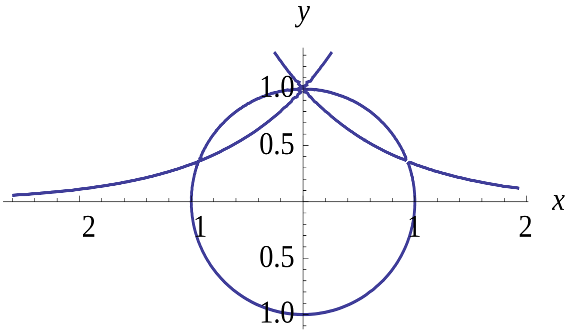

The equations for the graphs that were provided:

1. $(y - x^2 + 2)(x^2 + y^2 - 4) = 0$
2. $(y - x)(y + x)(x^2 + y^2 - 25) = 0$
3. $(y - 2^x)(y - 2^{-x})(y - 1) = 0$
4. $(y - x^3 + 1)(y + x^3 + 1)(x^2 + y^2 - 1) = 0$
5. $(2y + x)(2y - x)(y + x^2) = 0$
6. $(y - x^2 - 3)(y + x^2 + 3)(x^2 + y^2 - 9) = 0$

You might experiment with drawing a face or some other recognizable object. It is relatively easy to do this if you know how to construct the equations and graphs of ellipses. I have created two videos that explain this process. You may watch them here and here and learn ☺.

Hoping you enjoy your mathematics!

Graeme Henderson (June 2013)