

# How To Study Mathematics



Step-by-step instructions to help you set up an efficient study system so you can have a balanced and enjoyable lifestyle as well as greater success in your examinations.

# How to Study Mathematics



By Graeme Henderson

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## Effective Study Principles at Your Fingertips

If you are not going to study your mathematics then there is not much point learning it in the first place. It will not take long before you will forget what you have discovered (or been shown).

It is vital that you decide that you **WILL** study and it is also vital that you learn **HOW** to study effectively (as you certainly don't want to spend hours wasting your time either).

This very comprehensive summary has been prepared to help you achieve both these goals.

## Learning Comes Naturally to Us All

The first thing to realise is that you already know the keys for good study because you have been a good learner all your life! How do you know that you are a good learner? Because in your lifetime you have already learned many things:

- To be mobile
- To communicate
- To read
- To use a computer
- To use a phone
- To recall a vast number of memories, stories, jokes and songs
- To play at least one sport (probably a number of them)
- To play a musical instrument (possibly)
- To cook, dance, dress well or apply make-up well ...

and the list goes on and on!

You were born with **none** of the knowledge, memories or skills that you now have. That means that you have learned a vast quantity of "stuff" in your lifetime already!

It is almost certain that you know many things that I don't know and can do things that I can't do. You are much more clever than most people around you think you are. You are probably much more clever than **YOU** think you are!

### You are a good student/learner!

### And that means that you **ALREADY KNOW HOW TO STUDY!**

So, **HOW** did you learn (or study) all those things? Researchers identify five key ingredients for good learning. Because you have been so unbelievably successful at learning things over the years you will recognise all five of them. They will not come as a surprise to you. Here are the five key ingredients for good learning/study:

- Intention/importance
- Interest/variety
- Sense/meaning
- Value/relevance
- Rehearsal/repetition

Intention/importance is the first hurdle. To learn something well, you must think that it is **IMPORTANT** and **CHOOSE** to learn it. Oddly enough, this is much more important than **liking** what you learn.

The second key ingredient is interest and variety. What you learn must be stimulating and not boring. There must be variety and challenge and lots of small goals in your learning.

The third ingredient is sense and meaning. What you learn must have a structure to it and relate to things you already know.

Fourth is value and relevance. If some knowledge or a skill is of no immediate value to you, you will probably not bother to learn it.

The final key ingredient for good learning is rehearsal and repetition. Almost everything you have ever learned you learned through practice, and usually lots of it.

Just think about anything that you have learned during your lifetime and I am sure that you will see that you have used all five of these key ingredients during your learning process.

We will discuss all these ingredients later in this document when we learn about how your memory works and how you can make it work for you. Getting these five ingredients right is like turning your car's engine on. But first, we need to actually build the car.

The car that we are going to build is what I call your study system. The next few pages are devoted to helping you set up the four basic parts of a good study system. After you have these in place we will then come back to discussing these five ingredients and "switch your learning on."

## Four Steps to a Good Study System

### Step #1 ~ Turn Up (Record Your Time)

Good performance in any endeavour does not just happen. Whether you want to be a good musician, dancer, sportsperson, athlete, programmer, reader, scientist, historian, cyclist or swimmer you will need to invest time and effort in learning and developing the appropriate skills.

A good instructor/coach certainly helps, as does good equipment ... but let's think for a moment about YOU ... about good habits that YOU can develop.

- **Record the Time You Spend Studying**

The first habit that I suggest that you develop is easy one to put into place.

Simply record how much time you spend on all your academic activity outside of school hours. These are the hours when you can excel. Everyone else shares the same times at school. At home, however, you have a chance to be in control! So, keep a piece of scrap paper next to your desk or wherever you work. Each time you sit down and get back up again, record the time. At the end of each day, spend a few moments adding up the total amount of time that you spent on homework, assignments, reading set texts, personal study, etc. ... everything.

If you prefer to use a computer, use software that will allow you to clock on and off and will add up your time for you (some of these programs make it quite easy to keep track of how much time you spend on each subject if you wish).

At the end of each week (I suggest a week-end evening like Saturday night), add up your times for the week. That is all.

It is good to know how much time you are actually spending on your study. This in itself can give you a level of control and even satisfaction. If you can record this figure each week, you are establishing a very good habit.

- **Graph the Time You Spend Studying**

The second good habit that I encourage you to develop is to graph your total number of hours, week by week, on the one graph. This is such an easy thing to do as it will only take you about one minute per week to complete!

This graph will give you a **picture** of how you are spending your out-of-school time on academic development. Having and maintaining this graph is your first step towards maintaining an effective study system!

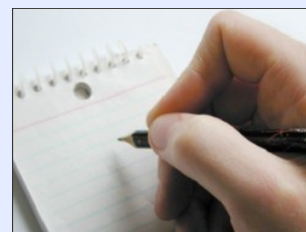
Keep this graph near your place of study (along with the scrap paper that you are using for recording times). You may make your own graph or download this [Excel Study Graph](#) (or [PDF Study Graph](#)) for your own use.

At this stage, it does not matter a great deal how many hours you are spending ... as long as your time is recorded. In time, as you become more motivated and invest more time in your personal study, this graph will help you see how much you have improved ... and that will be a reward in itself!



**You already know how to learn and have learned an incredible amount already!**

*Researchers identify five key ingredients for good learning. Because you have been so unbelievably successful at learning things over the years you will recognise all five of them.*



**Make a habit of recording all the time you spend on all schoolwork outside of school hours**

If your drive and motivation are poor and you need a bit of structure and discipline, then a time table is definitely for you!

*Almost all top students claim that their time management was one of the major reasons for their academic success.*



Preparing a good timetable is all about planning for lifestyle changes.

### Step #2 ~ Plan (Develop a Timetable)

Now that you have some idea of how much time you are actually spending on out-of-school work, it's time to look at your real week and plan what kind of time you can commit to. This is also a simple task.

- **Find Out How Much Time You Have Available (Unallocated Hours)**

Download and print a copy of your study timetable (you can open either a MS Word [Study Timetable](#) or a PDF [Study Timetable](#) ... but you can alter the Word document). Now spend a few minutes shading in all the time in the week that is already committed ... school, sleep, meals (including preparation and/or cleaning up), sport or music practice, work ... everything except for "free time" and time that you are currently studying. Next, add up all this remaining time (i.e. your free time and current study time).

This is a revealing figure. There are 168 hours in a week and most students will sleep for about 55-70 of them. That leaves about 100 conscious hours per week. How many of these are "uncommitted" for you?

This means that you now have two figures ... the actual number of hours that you spend on school-related work outside of school hours (from your study graph), and the number of unallocated hours during the week that are potentially available for study (that you have just worked out using the timetable). You now also have two decisions to make!

- **Choose the Number of Hours You Will Work**

Why is it important to be conscious of how many hours you work? Because it is the simplest measure and simplest way to monitor your study efforts! Every person wanting to excel in their field will either train for a given length of time or train through a set number of exercises (or laps, or routines). Since your homework and study exercises will vary enormously in scope and difficulty, it makes good sense to use the amount of time to keep track of your 'training.' Almost all top students claim that their time management was one of the major reasons for their academic success.

How many hours will you **choose** to commit to regular schoolwork each week? My suggestion is to set a goal that is achievable, not one that you are likely to fail to reach. For example, if you are currently

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spending three hours per week, you might consider five to seven hours as you would be unlikely to have the motivation or discipline to commit to 12-15 hours or more! If you are already spending 12 hours, then you might decide to set your goal at 15-20 hours. Remember that the more time you spend the greater your rewards may be. I will talk later about how to spend the time effectively and how to live a balanced life.

Make sure that you choose a goal before reading about Decision Number Two!

- **Make Lifestyle Choices to Reach Your Goal**

The second decision can be even more difficult to make. Unless you're very disciplined, my experience is that students new to the rigours of study will have difficulty committing to more than one third of their unallocated hours. Your body and mind need some time to rest.

So, take your study goal (hours per week) and multiply it by three (or two if you are more disciplined). If this figure is **less** than your number of unallocated hours per week, you should manage quite well!

If this figure is **greater** than your number of unallocated hours, then you may have a problem! You have three options:

- you could revise your study goal (you may be wanting to study for too many hours per week at this stage),
- you could try to be extraordinarily disciplined (it can work for some people), or
- you could make some lifestyle decisions and reduce the number of other commitments that you have each week in order to make time for study.

This will be your choice, your motivation, your discipline. Whatever you choose, you must be determined to stick with your choice and live with the consequences of your choice.

Bear in mind that you can always reevaluate your whole study plan at any time, but it is a good idea to try to stick to your new commitment for at least 4-6 weeks before your new habits become "part of you."

- **Draw Up a Study Timetable**

Finally, having made these two decisions, print a clean timetable and highlight the times each day when you will study. Make on your timetable matches the goal you have set. A word of advice ... DO NOT

allocate time that will conflict with activities you especially enjoy ... like a favourite television program! You know what will win!

It can also help to write the name of a particular subject in each hour or half hour slot so that, if you cannot think of anything to do at that time, you can simply study the subject written on your timetable.

Some students have good drive but do not like to be restricted to a time table. If this describes you, then just use the graph to monitor how you are going. However, if your drive and motivation are poor and you need a bit of structure and discipline, then the time table is definitely for you!

### • One Very Important Final Step

Now that you have decided that you are going to work towards a particular number of hours per week, and have set up a study timetable to help you get there, draw a horizontal line on your graph to indicate where your weekly goal is. As you record your study times during the next few weeks, you should see your columns hovering around

### Step #3 ~ Set a Goal (Goal Sheet)

Why do you want to study? Why are you reading this material? The fact that you have read this far means that you obviously have a desire to achieve something academically and it is this goal that will continue to drive and motivate you when the going gets tough. Here are some of the drives I have encountered in my students:

- the desire to enter a particular profession or course of study;
- the need to feel that they have given their (school) course their "best shot;"
- a desire to do well in whatever they do and not to do things half-heartedly;
- the need to work well for their own satisfaction;
- a desire to please their parents or a significant teacher;
- the fear of failure;
- the desire not to waste an opportunity;
- a very competitive spirit (sometimes against themselves, sometimes against peers or siblings);
- the desire to leave school on a "high note;"

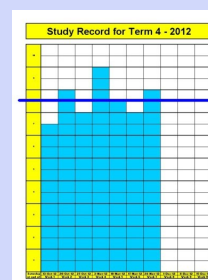
this goal line. If they fall behind for a few weeks, you know you will need to catch up. If you are exceeding your goal each week you can choose to take some time off and relax if you wish.

This graph is actually **more important** than your timetable! A study system should never be built upon a timetable only. **Time-tables always fail.** You will probably not have a single week where you are able to stick to it properly (due to sickness, accidents, visitors, excursions, etc.) but it forms a framework and a guide for you. It is the **graph** that keeps track of your fluctuating study from week-to-week.

Congratulations! This timetable and your graph recording the hours you spend studying are two of the four key things that you need in place for a good study system. One guides and the other monitors. The next two items need to address your motivation and to help guide you on your way!

Let's now look at the third part of your study system.

sure that the total number of hours per week



Draw a horizontal line to indicate your weekly goal.

I am aiming to study for an average of 12 hours per week for the next school term because, in the final exam,

**I want to beat Heidi, Chinwe, Hachiro and Hassim in mathematics.**

A goal sheet reminds you of the goal or reward that you are aiming for ...

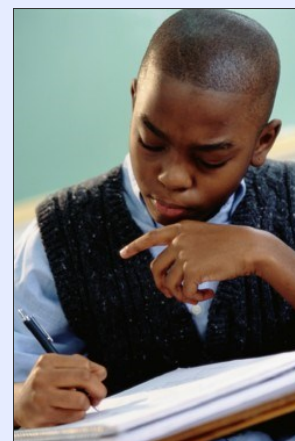
- the desire to keep their options open for when they leave school;
- a drive to ensure that thirteen years of school has not "gone to waste;"
- the drive to get into a particular job or career; and even
- the sheer love of learning.

### • Establish a Goal and Write it Down

Try to identify what is motivating you to study. Why did you begin reading about "How to Study?" Your motivation may be one of the ones listed above ... it may be something quite personal.

*Your time graph is actually more important than your timetable!*

*Timetables always fail.*



Inspire yourself! Take time to set an achievable goal (and possibly a reward).



Give yourself a treat ...

It feels amazingly good to reach your goal!

*He confessed to me that there were times that he felt like giving up but each time he looked at his goal sheet he was driven to continue.*



Rewards do not have to be expensive.

Ideally, your goal should be **measurable**. That is, you should know when and where you reach it. For example, if you desire to get into a particular university course with a high entry level, you probably have a good idea what results you will need to reach that level. You may even be aware that your school typically has, say, three students that reach that level, so you might also think in terms of being in the first three places in your final exams.

It is possible that your goal may not be measurable. It might be as simple as putting forward your best effort ... much as your goal may be simply to run each day regardless of where you run or for how long. In this case you might think of writing something like "Best Effort = 10 hours per week."

#### **What to do if You Have Difficulty Setting a Worthwhile Goal**

Reaching your own goals is often sufficient reward in itself (think of the satisfaction of climbing to the top of Everest and just knowing that you did it). Ideally, this kind of satisfaction should be what drives us all. However, it is not always like this in the real world.

If you have real difficulty identifying a goal that motivates you sufficiently, you may consider creating a **reward** that you will receive if you simply stick to your study plan (average number of hours per week) for a given period of time.

I have at times encouraged students to sit and negotiate with their parents or guardians. If they want you to do well too (most do), they may agree to take you somewhere special or buy/make you something special or do something special with you when you reach your goal. In this case, your goal will be to do a certain number of hours each week for an agreed number of weeks. Because they will be making a sacrifice, they must be able to check that you actually do the work and it is very important that they agree NOT to give you the reward if you fail to reach the target! For your part, it is important that the reward is something worth working for.

As an extreme example, some years ago I taught one young man (about 14-15 years

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of age) who had never studied at all and he was suddenly inspired to get started. After negotiating with his (not wealthy) parents, they agreed to buy him a trail bike if he could complete an average of ten hours "study" per week for two school terms (half a year). This was a very expensive gift but they wanted him to do well and were not about to give him such a gift for just one or two weeks' work!

I can still remember his excitement when he sought me out later in the year to tell me that he now had a trail bike! Not only that but, because he had been practising such good study habits for so many weeks, I believe he continued to study well for the remainder of his school career! He confessed to me that there were times that he felt like giving up but each time he looked at his goal sheet he was driven to continue.

I might mention that an added bonus is that, often, house life and relationship with parents/guardians dramatically improves too (more respect, more trust ... I have watched it happen)!

Make sure that whatever goal/reward you have chosen is worth working for and will make you feel good when you get there.

#### **Write/Draw Your Goal and Display it Prominently**

Now that you have chosen/negotiated your goal/reward, get a sheet of paper and write or draw your goal on it!

For most people, a vague goal is no real motivator, and you want/need something that will spur you on when you don't feel like studying. That is what this sheet of paper will do. Place it near your place of study so that it is quite visible.

By now you should have three sheets of paper in place:

1. a goal that you are aiming for,
2. a time table that is basically a plan to help you get there, and
3. a time graph which is a very visible record of your progress towards your goal.

There is one more item that you need. This last item is one that is not used very well by many students but it is absolutely essential for good learning as I will explain later.

Just one more step and you will have your study plan in place!

#### Step #4 ~ Organise (Use a Diary)

A large part of the rationale for studying is to ensure that things you have already learned are transferred from your short term memory to your long term memory. A wonderful tool to help you achieve this is a diary or calendar. It does not matter whether you purchase an inexpensive diary or whether you use your phone or computer, it is simply important that you are able to keep track of your study/revision.

#### Home Exercise: Learn to Use a Diary

So often I hear students say that they could do the work in class but had forgotten what to do by the time they were tested on it a few weeks later. This is not because they are bad at mathematics! It is because they have not learned how to move their knowledge and skills to their long term memory. If I asked you to complete a test where you had to recognise colours or name basic shapes or make simple additions like  $2 + 3$ , you would do very well indeed. This is because you do not have to cram all this into your short term memory before such a test. In fact, I suspect that you would not even study for such a test! All these things were committed to your long term memory many years ago. How? By constant repetition over a long period of time. Wouldn't it be nice to know your area and volume work, or trigonometry, or calculus, or statistics so well that you could recall the information and the skills months and years later just as easily?

There is no real secret about how to achieve this. I am sure that you have heard it over and over again from teachers and parents. The secret is practice ... repetition! The problem is that repetition alone does not work. I am going to talk about using your diary first, but the other things that I will share here may make all the difference between your success and failure.

First, the diary! I hope you will be pleased to know that there are smart ways to practise/repeat your studies. You could try to revise/cram just before each test, but you know that does not really work well. Shortly after the test you have forgotten it again. You could try to repeat volumes of work for hours and hours, week after week ... but that may not work well either (it can be very inefficient). Ideally you want to do just enough work at just the right time to reap the most benefit from it. Here's how!

After each mathematics study session I want you to do two things ... ask three questions and write a conclusion. The questions are these:

1. Do I understand the work well enough?
2. Can I solve the problems quickly enough?
3. How long do I think I can go before I risk forgetting this work?

Before information or skills can be stored in your long term memory, you must **understand** it. This sounds so simple and obvious, but so many students stop working before they have achieved understanding (or they do their homework with distractions so they do not concentrate on developing understanding in the first place). It is also vital that you develop **speed** when solving problems. Almost every test given in schools is timed. That means that you race against the clock. I have watched students do very good work but take five minutes to solve a two minute question in tests. If your answer to either of the first two questions is "No" then you need to speak with your teacher/tutor and schedule some more revision before you have too much new work added (i.e. usually within 24 hours).

If your speed and understanding are good, then your homework has achieved its purpose! In order not to waste all this and have to learn the same material all over again some time in the future, you ask the third question ("How long do I think I can go before I risk forgetting this work?"). Place a note in your diary on an appropriate day to revise the information or complete a few questions from that part of your textbook.

Of course, to benefit from this, you need to check your diary each day! If you have timed your entry well, you will still remember the work and be able to do it quickly when your diary reminds you to do the revision. After this, ask the same question and make another entry in your diary. You might now feel that you could go for a month or more before revising. Each time you revisit the work the information/skill is being embedded in your long term memory.

Let's give an example. At school, you have just learned how to find volumes of cones and cylinders. That evening, you complete some homework ... let's say there were ten questions given. When you have finished that exercise, let's assume that you understood the work and were pretty happy with



An electronic or paper diary is one of the best tools you could possibly use to help place knowledge and skills in your long term memory.

*Remember to ask yourself the three questions (see left) after every homework and study session.*



Timing your homework or study is a great way to build concentration, help time pass quickly, and get very fast at your mathematics.



**Distributed practice** (revision spread over a period of time) is essential for storing things in your long term memory.

*Your tools for an effective study system are: a goal sheet, a time table, a time graph and a diary.*



**Take time to make friends with your memory and learn how it works.**

your working speed. Now ask the question, “How long do I think I can go before I risk forgetting this?” Perhaps you may be confident and feel that you could safely remember it for a week or so, but you are fairly sure that you would struggle to remember it after three weeks. So, you find a day in your diary two weeks from now and make a note to do, say, eight questions from this particular exercise during that evening.

Two weeks later, when you check your diary, you see the note and do the work. Two things could happen: you may remember it all or you may not. If you do not, the refreshing exercise will be of great value. After completing those questions, you now ask yourself the same question. This time you may feel that you could go a month or more. Each time you repeat the process the time should lengthen. My experience is that, within about 2-5 cycles, you will be confident of going for at least 6-12 months without needing to refresh your memory. This is a sure sign that the knowledge and/or skills are now being es-

## How to Remember Things for a Long Time

It is all very well having a study system in place. It is now important to make sure that it actually works! One of the main goals of your study is to actually **remember** the material when you need to use it (during exams, for example).

This means that you need to understand something about how your memory works. Fortunately, researchers have been learning a lot about how our memory works (of course, they use mathematics to do this!). They have learned that all of us have three types of memory that use different parts of our brain.

### First Type of Memory ~ Immediate

You have an *immediate memory* that can store just a few items for up to about half a minute. After this time, unless the information is very important, it is forgotten. Your immediate memory temporarily stores things that you notice or find interesting. For example, if you continue to breathe normally through your nose you will be aware of the air rushing over your upper lip as you exhale. This sensation has been there ever since you were born but you only noticed it now when it was drawn

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established in your long term memory! It also means that you do not have to cram this material when exams approach.

Memory researchers describe doing a lot of work at one time as *massed practice*. This is an excellent way to learn quickly but the lessons can be forgotten quickly as well (especially if it did not mean much to you). Spreading your learning out using a diary is what researchers call *distributed practice*. Both are good and useful, but it is particularly distributed practice (sustained/repeated practice over time) that produces good learning.

So, you now have all four items in place for your sound study system ... your goal sheet, time table, time graph and diary. Learn to use them well and you should have a flexible but strong study system in place.

There are other things for us to discuss, but the effective use of these four items forms the foundation for anything else that I am going to share.

to your attention, so it was temporarily important to you.

You use this same memory when the teacher says, “Open your text books to page 127 and get your protractor ready.” You only have to remember this information long enough to do the task. If you were asked a few minutes later what page you were working on, it is likely that you will have forgotten as it was not particularly important that you remember this fact.

So, information not only has to be important or relevant to even enter your immediate memory, it also has to be important to you in order for your brain to want to keep it.

Notice that does not necessarily have to be *interesting* or even something that you like. You do not even have to like the person drawing your attention to whatever is of interest (we will discuss these issues later)! Simply put, to register something in your immediate memory, you must consider it **important** and **intend** (or choose) to learn it.

Notice that importance and intention are not passive states; they are attitudes that you

choose to adopt. Therefore, the word that I want you to associate with immediate memory is **CHOICE**. You can choose whether or not you intend to listen and pay attention. You can choose whether you'll consider something as being important for you and whether you will pay attention to it. In other words, you can **CHOOSE** whether or not you want something to enter your immediate memory and enter the learning process.

If you choose not to pay attention in class or not to pay attention when completing your homework or study, for example, your chances of successful learning are very low indeed.

To be a successful student, you need to ask the following kinds of questions to make the best use of your immediate memory: Will I choose to learn? Will I pay attention in class? Will I work with as few distractions as possible? Will I choose to regard the material as important and intend to learn it?

Students who pay attention during lessons and pay attention to (and focus on) their homework and study have to exert far less effort to remember than those who don't pay attention.

### Second Type of Memory ~ Working

The second kind of memory is called *working memory*. It is possible to keep really important items in this memory for up to a few days (e.g. if you need to hand your assignment in on Friday), but you can usually only maintain each thing in working memory for 10-20 minutes unless something different happens to make the item new and interesting again. If boredom or fatigue sets in it is likely that your working memory will dump an item in favour of something else.

Most of us can only keep about seven items in our working memory at any time. This means that you must be careful not to clutter your working memory. Effective study is done when you concentrate on fewer things, especially when you can create variety during that time. Do not try to multi-task ... watching television, chatting on FaceBook, texting friends, and listening to favourite song while trying to do your mathematics homework ... especially if you have different kinds of questions to solve.

It is also important that you change your approach to your work regularly ... summa-

rising for a while, solving some questions a different way, checking answers by substituting, trying to do a number of questions at high speed, etc. Create lots of small goals and challenges as you learn. For example, if you've got ten questions to do for homework, time yourself on the first two and race the clock for the remaining eight. This variety keeps your working memory on track.

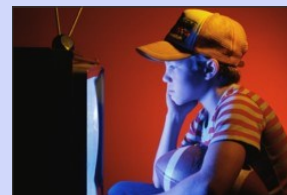
Working memory is great for rote rehearsal of things like times tables and facts and definitions, especially if there is some sense of excitement, like racing the clock or tension if you are being quizzed by someone. It is also very good for what researchers call 'elaborative rehearsal' where you spend enough time with the material to start to see its connection/relationships with other things that you have learned. When you do homework you should be looking for links like this because each connection greatly enhances your chances of remembering the material. You might be studying some trigonometry, for example, and suddenly realise how it is connected to Pythagoras' Theorem. These links also help you solve higher order problems that only the better students can manage.

To transfer things efficiently from your immediate memory to your working memory you must find them **interesting** and you should ideally use lots of **variety** in your study. Boredom and fatigue almost guarantee that this learning step will not be achieved easily. It is up to you to create (or find) interest and variety if your textbook and/or teacher does not. The easiest way to do this is to make contact with other people (students, teachers, relative or friends) who are motivated about learning whatever you wish to learn. I will have more to say about this under the heading "Study-Enhancing Lifestyle Choices" and under "Condition Two" of the Long Term Memory section on the next page.

No matter what you want to learn (how to strip a car engine, how to play a sport, or how to do quadratic equations), try to spend time in the company of interesting, enthusiastic and knowledgeable people! It makes sense, doesn't it?

### Third Type of Memory ~ Long Term

The third kind of memory is called *long term memory*. I am sure you have heard of this memory. Ideally, this is where all of your useful information should be. Imagine

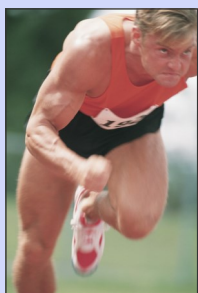


**Electronic distractions may not stop you from getting homework done, but they ruin your chances of remembering the work.**

*To learn well, meet the three conditions for long term memory: Sense/ Meaning, Value/Relevance, and Rehearsal/Repetition.*

7	X	1	=	7
7	X	2	=	14
7	X	3	=	21
7	X	4	=	28
7	X	5	=	35
7	X	6	=	42
7	X	7	=	49
7	X	8	=	56
7	X	9	=	63
7	X	10	=	70

**Variety is needed to keep your working memory focused on a task like learning tables. Change your approach often. Be creative!**



**We all know that determined practice leads to success.**

*Attitude makes a huge difference. Mix with the best and most enthusiastic mathematicians and learn from them . . . face-to-face, through books, and on-line.*



**You do lots of things during your school years that you may not be doing when you are forty. This does not mean that they are bad or wrong or a waste of time! This is also true of mathematics.**

if you could remember everything that you wished to remember without effort! Exams requiring only knowledge recall and basic skills would be a dream! Of course, in reality, no one can remember things without effort. Knowing this, your study goal is to store things in your long term memory as efficiently as possible.

To get your information or skills from working memory to long term memory in an effective way, at least three significant things must be in place. If you have not satisfied these three conditions it is quite likely that you will forget whatever you have just been doing.

- **Condition #1 for Long-Term Memory ~ Sense/Meaning**

It is absolutely vital that you understand the material. If you simply use the formula and follow the pattern that the teacher explained and especially if you do it in a distracted way (distracting environment) it is likely that you will forget all that you have done. This is a waste.

For example, if you had to memorise a few sentences in a language that was foreign to you, you would only see strange symbols and hear strange sounds. Once the meaning is explained, remembering becomes so much easier.

If you can say after a mathematics lesson or after your homework has been completed, **“I don’t understand,”** this is a huge warning light to you that this condition has not been met. Please seek help in understanding. Participate fully in class (so your time there is not wasted), ask your teacher or tutor, ask friends, read books, watch [YouTube](#) videos — do whatever you need to in order to understand.

This should not cause embarrassment but, unfortunately, you and I live in a world of bullying and shaming people, so you may have to ask your teacher for help outside class time (although it is usually better if you can ask questions during the lesson).

I remember one brave girl that I had in one of my classes some years ago. During just one lesson, she answered thirteen of my questions in a row (yes, I counted them) and got every answer wrong! I had to defend her against some in the class who wanted to laugh at her and then took the time to explain to her and the class what the correct answer was, and why. I am delighted to say that, for that topic, she rose from

## HOW TO STUDY MATHEMATICS

the bottom of the class to very near the top! I was so impressed! For this particular topic, because she bravely engaged with the lessons, she satisfied this condition . . . she **understood!** If only the rest of the class had engaged in the same way.

- **Condition #2 for Long-Term Memory ~ Value/Relevance**

This question of value/relevance is a difficult one! Unfortunately, for you to remember work well you must actually want to remember it! It must have some value for you. I am sure that you are familiar with the warning questions: “Why do we have to study this?” “When will I ever use this?” If you ask these questions, then this condition has not been satisfied for you and it is highly likely that your study will be ineffective and that you will struggle with your mathematics.

I am going to make some comments and observations here but my answers may not satisfy you (however, I hope they help).

First of all I want to ask a serious but difficult question of you. If you found mathematics easier to do and were succeeding at it, would your attitude be different? If you think it might be, then satisfying this condition may be easier for you. If you think you would still hate it, you have some real soul-searching to do if you are to satisfy this condition and succeed in the subject even if you do not like it.

**Will you ever use this mathematics?** I have my own reasons for studying mathematics but I am aware that they are mine, not yours. In truth, you may never need to recall trigonometry or calculus or quadratic equations for the rest of your life (after you leave school). But there is so much that you already know that you may never use again. Some of you may do gymnastics or skateboard or surf or play a musical instrument — in fact, you might be doing all kinds of things at the moment that you may never do after you leave school (you may of course, but many things will stop as other things in life take over). If you have invested time in learning something that you never use or do again after you leave school, has it been a waste?

In the past I have played a musical instrument, sailed, engaged in track and field events in athletics, been a trampolinist and studied a martial art, and played around 8-12 sports that I no longer play. All of these experiences were of value. When I see

them on television, on YouTube, in the news, or am chatting about them with people, I have a much wider appreciation of life and what is being portrayed or discussed. Even though I am no artist I am glad that I learned some art history and appreciation.

In sharing this, I would like to suggest to you that there may be aspects of mathematics that you may forget but will leave you with an appreciation for other things in life. For example, after having learned some trigonometry, you may find that you understand what goes on when someone surveys the land that you buy or how the ships or planes that you travel on navigate from place to place, or how mapping and GPS works. You may never wish to work in those areas, but knowing about them may not be a total waste.

I know what I have said may not convince you, but it may help a little. In either case, the bigger question is probably the other one!

**Why do we have to study this?** The answer that many teachers give to this question is that it will be in the exams. You and I both know that this does not motivate you enough. All it may do is add to your stress levels and raise your anxiety levels but it does not really make the work valuable TO YOU!

Now you are going to hear something that is a bit confrontational — who is it that ultimately has to answer this question? Your teacher may attempt to answer it, but ultimately YOU are the one who has to find an answer that you are satisfied with.

If you went to your local sports coach (whatever sport you learn) or your music teacher, for example, and asked why you had to practise/study some skill, the answers would always be given in the context of succeeding at what you are doing. Why do you learn layup shots in basketball? So you can increase your score rate in games. Why do you practise scales in music? Because you ultimately become a much better musician if you have that skill deeply embedded in you. The honest answer to why you are learning different things at school is to make you a better mathematician. And here is the problem — you **chose** to learn that sport or instrument but doing mathematics at school is **compulsory**!

How can you overcome this problem? The fact is simple really, but not necessarily pleasant for you to hear. You must learn to somehow like or enjoy the subject or forever struggle with it (or give up on it). There it is in its simplicity.

Now I need to ask you a profound and very serious question. Are you prepared to find mathematics interesting or is your hatred of it so deep-seated by now that you cannot even comprehend that? The sad fact is that, if you cannot **want** to study the subject, you are going to always find it quite difficult to commit material to long term memory. Please take some time during the next few hours, days or weeks to really think this through.

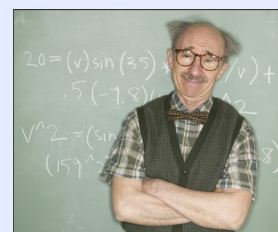
If you are prepared to have your attitude changed, then the answer is relatively simple to say but may be difficult to achieve. Listen carefully. I have never skateboarded. To be honest, I have no great desire to (if you are in high school I am probably about three or four times your age). If I wanted to learn however, I would seek out a really good and inspiring skateboarder and ask him/her to teach me (one of my friends is very talented and has been deeply involved with local skateboarding for years).

If you want to feel better about mathematics you need to find others who are talented and enthusiastic about it and who can teach you. For some of you this may be your teacher (if they are talented, enthusiastic and approachable, you may need to have a serious chat with them outside class time). Sadly, some of you will have really dull, boring and unenthusiastic teachers. Try to stay on good terms with them (i.e. don't aggravate them), but you will not be able to rely on them to inspire you in any way. You could try other teachers, good tutors, a mathematics club, some of your peers, as long as you can have a good laugh and some fun as you learn. Search [YouTube](#) for videos, visit your school or local library and look through the mathematics section for something that interests you (codes, puzzles, patterns and designs in art, mathematics of music and all kinds of things). Ask your parents or other adults if they know of anyone in the local area who is enthusiastic about mathematics and uses it in some interesting way (electronics, or building or radio control etc.). Keep seeking and asking around. If you were wanting to get good at some sport I am sure that you would hunt around for the best coach you



Discussing your mathematics can really help.

*To memorise facts well your brain needs 'to engage in elaborative rehearsal.'*  
*Doing challenge questions and puzzles helps your brain make more connections between new facts and those that you already know.*

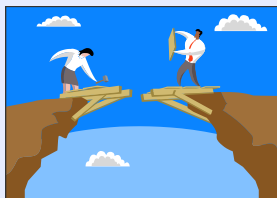


Your teacher may or may not be inspiring. If he/she is not helpful, find an inspiring mentor somewhere else!



Discussion can be a very effective means of revision for many students.

*One of the best tools that you can use to monitor your rehearsal and repetition (to enhance your long term memory) is your diary.*



Elaborative rehearsal is experimenting and playing 'what if' games. This means that you are making connections between facts that you are learning and facts that you already know.

could find. Do the same for mathematics and, hopefully, the people and resources that you find will be able to help and inspire you so that your school lessons are not all you experience of the subject.

If you can answer this question for yourself and say that you are learning mathematics because it is interesting or stimulating or challenging or useful or even because you like it, your chances of storing volumes of material in your long term memory have just increased massively!

### • Condition #3 for Long-Term Memory ~ Rehearsal/Repetition

If the first two conditions have been fulfilled, this one is rather easier. I have described in the last section (Four Steps to a Good Study System) how to use your diary effectively to study things at regular intervals in order to remember them well. Please take what I shared to heart and use your diary as an effective study tool. Use variety. Remember that revision can be oral (speaking it out), aural (listening to a recording), relational (being quizzed by someone or discussing the topic with them — some students use their parents or siblings or friends and others manage to do their homework while using Facebook to discuss it with friends at the same time), mental, on paper, etc.

One of the best tools that you can use to monitor your rehearsal and repetition (to enhance your long term memory) is your diary. I have discussed its use in an earlier section and explained how it can be used to keep track of your massed practice and distributed practice. Please reread that section or view the video to remind yourself of its importance.

Note that rehearsal and repetition requires **time**. This is why students who do not devote sufficient time to their learning must always struggle. If you have not already read or listened to the earlier section entitled "Setting Up Your Study System ~ Turn Up," please take the time to familiarise yourself with the material there. It explains how to set time aside for your study and monitor it.

Rehearsal and repetition is incredibly important for good learning, that is, establishing things in your long term memory. It is so important that I am now going to devote the next two sections to discussing how you

## HOW TO STUDY MATHEMATICS

can learn facts and skills through rehearsal and repetition.

### How to Keep and Secure Things in Long Term Memory

Broadly speaking, there are two kinds of things stored in long term memory, facts/events and skills.

#### • How to Deal With Facts

Facts are stored in what researchers call *declarative memory* and require conscious processing in the frontal lobe of the brain. Once facts (**what** and **why**) have been introduced to long term memory, they are retained best if you can engage in what is called "elaborative rehearsal" of them. This means that each time you think of these things, you make more connections between them and other things that you know. You do this by doing extra reading, solving extra problems, discussing them with other people and being creative and analytical in the way you think about the facts. For example, if you wish to remember Pythagoras' Theorem better, you could find a book or two in the local library that had something to say about it, or go searching the Internet for something interesting, or find a different textbook or past exam papers or a puzzle book with different kinds of questions in it, or chat with your teacher/tutor about other ways of understanding or using the theorem, or use it to do something interesting like erect a bridge over a creek or building a tower or some such thing (be creative but safe and have fun ... this is where 'nerds' get up to mischief). You should be trying to develop new ways of looking at the same thing and new ways of analysing the same situations. I actually like reading history and finding how mathematics was used through history. This helps me learn about its connection with art and hygiene and food and discovery and explosives and medicine and so on — I think you get the idea.

It is a huge task but my plans are to gradually add suggestions and material like this to the [CrystalClearMaths](#) website. Feel free to offer your suggestions via our [Feedback](#) page!

#### • How to Deal With Skills

Skills are stored in what researchers call *procedural memory*. This is where you remember **how** to do things that require a

set of automatic steps. These include motor skills like walking, kicking balls, swimming, gymnastics etc. When you see a Olympic divers dive from the 10 metre tower, they are using procedural memory. They practise each skill until it can be done efficiently and almost without conscious thought. Some researchers describe this as moving skills from being *reflective* (requiring thought) to *reflexive* (not requiring conscious thought). All skills are perfected by rote rehearsal (lots of practice). A large part of all mathematical skills should be stored in procedural memory, i.e. you need to practise them until you can do them accurately and efficiently and speedily. Believe it or not, for a good mathematician, there is very little conscious effort in solving a relatively complex linear equation involving fractions, for example, because the skill has been practised and refined.

The key here is the same one that will apply to any skill that you want to master — it is not simply **practice** that makes perfect, all that practice does is make things permanent! The more accurate saying is that **perfect practice makes perfect** (so said Vince Lombardi, a famous US football coach)! In other words, whenever and whatever you practice, try to do it properly and well without taking shortcuts or being sloppy about it. In mathematics, this means practicing setting your work out properly each time.

#### Some Suggestions Concerning Practice/Rehearsal/Repetition/Study

- Learn what is “perfect” first and then practise it. Avoid practicing things you do not fully understand. All you are likely to succeed in doing is making bad habits permanent and then they are very hard to change. You should therefore listen carefully in class and learn to set your work out clearly and in distinct

steps (procedures). In other words, learn things properly in the first place. Correcting bad habits is difficult and demoralising.

- Learn from an expert rather than on your own (unless you are very good at mathematics). This is why I do not agree with students learning their own method for solving arithmetic problems until they have been taught a useful algorithm (which they have mastered) first. They are then free to explore all kinds of other options. Unfortunately, many students learn bad habits without guidance. It is a bit like asking someone to learn to snow ski by going up the chair lift for the first time and ‘working it out on the way down.’ It is much better to receive instruction and learn good habits early.
- Remember that you need to understand (have sense/meaning) and be motivated (have value/relevance) in order to learn well. If you truly understand your work you should be able to analyse new situations to see how to apply your current knowledge and should be able to analyse your work to see where you could improve upon it.

I hope this helps you become more aware of how you learn and helps you plan to study accordingly.

If you want to read more about how to learn mathematics let me recommend "How the Brain Learns Mathematics" by David A Sousa. You might ask your school librarian to purchase a copy from [Amazon](#) or [The Book Depository](#). The beginning of the book discusses the results of recent research into how the brain learns mathematics. The remainder of the book is addressed particularly to teachers, explaining how they can use this understanding to prepare better lessons and help students of different ages learn better.



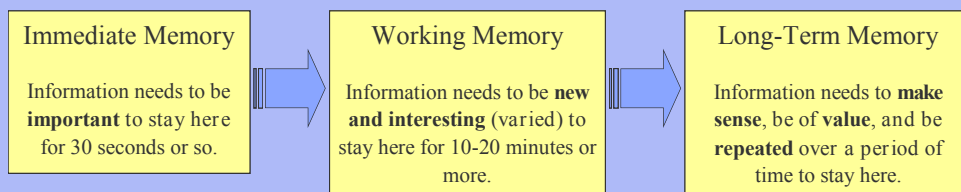
Skills are stored in 'procedural memory' through lots of practice.

*Practice only makes 'permanent.'*

*Perfect practice makes perfect!*



Regardless of whether you feel 'naturally gifted' or not, always try to learn from experts.



## The Process of Learning

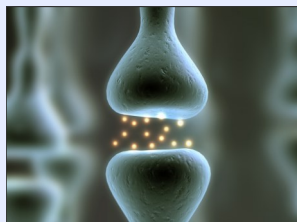


**Start studying now.**

**First steps give a real feeling of achievement.**

*Try to do quality work for each task.*

*Success is the sum of little things done well.*



**There are many factors that influence your memory. Learn what works for you.**

## Seven Study Attitudes Worth Cultivating

### Principles/Attitudes to Adopt for Success

1. Start to study! Do it now! Even smart people need to practise in order to achieve. It is a myth that they don't. Olympic swimmers spend hours each day swimming laps, concert pianists practise their scales (and other exercises) diligently ... it does not matter how good you are or what skill you want to develop, practice is essential for excellence and success.
2. Start earlier rather than later. Don't procrastinate. Is isn't only in Aesop's Fables that tortoises catch up with hares. I have observed that, quite often, talented students who get good results based on their ability, but who do not develop a good study pattern, are significantly outperformed during the last two years of school by 'plodders' (students with less ability but who study methodically). The longer you procrastinate and put it off, the less likely it will be that you will ever get started. So, don't just decide that it is a good idea to study — do something about it and start today!
3. Focus on everything that you do — whether it's homework or assignments or study — try to do it all well. Be determined to produce quality work. Remember that practice only makes permanent. Whether you practice sloppy or excellent work, the practice itself will begin to make it habitual. So, make a habit of producing quality work. This should lead also to some measure of success for you and your enjoyment of success can mean that later work doesn't seem so much of an effort. Remember it's "perfect practice that makes perfect" and "success is the sum of little things done well."
4. Unless doing oral drill work, always show your working and always practise setting your work out neatly and clearly. When you're first working in any new topic you should be developing thorough understanding (and developing a procedure for setting your work out and answering questions) that improves your success rate and reduces the possibility of your making careless errors. Determine to make this a habit and it will not fail you.
5. Drill work! Choose to do as many questions as you can to build understanding and speed. Once you have a well understood procedure in place for setting your work out clearly and answering questions, try to do a lot of your mathematics study using oral drill work, that is, 'mentally.' Having someone, or a computer program, to check your answers and give you immediate feedback allows you to concentrate on building speed without having to practice all that writing. In this way you can often complete questions at five and even ten times the rate you that would complete them when writing them down. You might also be surprised at how rapidly you can solve some problems in your head, with practice.
6. If you want to become a really first-rate mathematics student, never let a question go by that you cannot do or solve. These questions provide your best learning opportunities (and questions like them could appear in your examination paper). This is incredibly valuable time and most students never, ever, get to benefit from it. When you spend a long period of time on one question or problem, you'll be trying out all kinds of approaches and practicing all kinds of mathematical skills. This attitude makes a good student into a really top student!
7. Experiment with a variety of ways of immersing yourself in the learning experience and major on the ones that work for you. Such methods would include writing notes, reading out loud, listening to sound files, watching videos, discussing with others, teaching others, practising a skill on paper, reciting while walking, composing a song or poem, getting someone to check you orally, etc. Have fun thinking up, and trying out, new and creative ways of learning. Be very wary of all those nice graphs that tell you you only remember 10% or what you read etc. The 'research' behind those figures is bogus at best (see the results of [Will Thalheimer's research](#)). The truth is that some people learn better by reading and others learn better in other ways and that there are many factors that influence your memory. Learn what works for you and your study will be much more effective

## Seven Ways to Improve Your Study Environment

Find a solution that suits you and your lifestyle, but if you want to do what most top students do, try the following:

1. **Study without distractions** in a separate room at home, at school after school, or at a local library. Most students prefer a separate room at home where they can set things out the way they want and leave things as they are for when they return. Those who study at school or the library usually do so if the home environment is too distracting. Although the vast majority of students study almost exclusively at home, some study in their bedrooms by necessity or from preference while others like to keep their study area separate from where they sleep, relax and socialise. Give some thought to building your daily routine around when you can find such a place and a time free of distractions. Your study will be that much more effective.
2. **Use a comfortable table and chair.** Posture is incredibly important when studying (or in class). Slouching in your chair or slumping over the desk while you're studying has been shown to restrict free breathing and to decrease your alertness — and alertness is vital because it allows you to achieve better results in a shorter period of time. Studies show that simply improving your posture can make you more alert and increase your exam results noticeably! Make sure that your chair gives good lower back support and that your table and your chair are at a good working height. Ideally, you should be able to put your feet flat on the floor or on some footrest, and the table should be just above elbow height. It is better to use a larger table (if possible) because it allows you to spread your work out.
3. **Control your exposure to electronic distractions.** Most top students control their use of the computer and social media using a range of methods. Some choose to have the computer in another room so they have to change rooms to look things up; some turn off their mobile phones while they are studying; and some even resort to having friends or family change their passwords for Facebook etc. each term or the entire final year of school. Hardly any top students

study near the television. I have known some students who have used Facebook or Skype very successfully while doing their homework and discussing their work with friends. That may work for you also, as long as you're disciplined enough to resist the temptation to simply socialise. If you're determined to become a top student make sure that you have control over your social media!

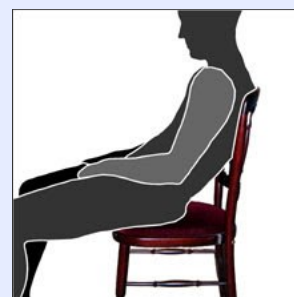
4. **Study alone!** Most top students do their serious study alone for the majority of the time. Some top students also spend time with a 'study buddy' or a small group of similarly minded friends on a semi-regular basis and find that having discussions with these friends helps a lot.
5. **Study quietly.** Most top students study in silence. A few will listen to music but almost all of them will turn the music off when they have serious study or memorising to do. Of those who listen to music or are near the TV for 'lighter' work, the majority will only listen to instrumental music (for which there are no words or lyrics) if they are having to read or study text-based subjects. Some will listen to songs while they study mathematics or do art work etc. I strongly recommend that you not listen to songs while studying mathematics and preferably study in silence. From my discussing this with them, and reading interviews with them as well, I believe that the majority of top students would agree.
6. Most serious students like an **airy (open) and naturally lit room**. Be very careful with lighting at night. Some students, particularly those with certain forms of dyslexia, develop significant eye strain and tiredness when working under fluorescent lights. Try to ensure that the lighting comes from your left if you are right-handed or from your right if you are left-handed. This reduces the size and intensity of shadows in your work area and helps reduce eye strain.
7. Take **regular breaks**. Your body and your mind need regular change. Take time to rest your eyes, **exercise and stretch** a little, **socialise** a little, or simply **rest and relax** before getting back into study.



**Make sure that you control your social media.**

*Give some thought to building your daily routine around when you can find a place and a time free of distractions.*

*Your study will be that much more effective.*



**Be careful with your posture when studying (or in class).**

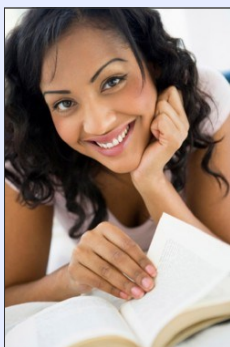
**Studies show that simply improving your posture can make you more alert and increase your exam results noticeably!**



Most of your quality study will be done while you are alone.

Don't neglect studying occasionally with a "study buddy" or a group of similarly minded friends.

*Base your evening routine on "homework, review, preview, plan" (see right) ... and learn how to do each of these skills well.*



Taking just five minutes to pre-read your texts each night can make a huge difference to how much of the work you will remember.

## Seven Steps to Follow Every Evening

Ideally, try to complete the following steps each day:

1. do all your **required home work** (remember to check it off in your diary as you complete it);
2. remember, after each item of homework or study has been completed, to **ask yourself the three key questions** ('How well do I know this?' 'Can I do the work fast enough?' and 'How long before I might forget this work?') ... and then, based on your answers, either do a little more work immediately or make an entry in your diary to revise it on an appropriate future date;
3. **complete some work on outstanding assignments** or compulsory reading so you can keep on target for completion by the due date;
4. while it is open, **check your diary** for any revision items that you have scheduled from previous study efforts and either complete them or postpone them to the following day (remember, when you complete these items, to ask yourself the three key questions again and reschedule their further revision for some time in the future ... hopefully, after a longer time interval);
5. **start another project** by choosing a subject and some new topic or skill to summarise or revise ... and then do so,

remembering to use a diary entry to keep track of this work also;

6. if you have a few spare minutes, check your school time table for the next day and **pre-read the texts** to gain some idea of what each teacher is likely to explain (a minute or two doing this on the night prior to a lesson can make your retention of the lesson material much greater on the day ... a very effective use of time), and
7. **write out a 'to do' list** for the next day ... rank the items in order of importance, and try to complete as many as possible during the day.

When all this is in place, you can sleep well, knowing that you are organised and in control of your learning.

This seven step system (based on homework, review, preview, plan) provides a method for you to keep on top of your work.

Be prepared to experiment to find your own rhythm. Some students work best on 20-30 minute bursts with 10 minute breaks in between. Others thrive on hour-long sessions with longer or shorter breaks. This is a very individual thing and will often vary with how tired or stressed you are. Get to know yourself and adjust each evening accordingly.

By now you should have your study plan in place and have worked on your mental attitudes and your physical posture as well as a routine for each evening. These are all good and important things but, when you actually sit down to study mathematics, what should you do?

That is what the next section is about! Please read it carefully.

## Downloadable Summaries

If you wish to download one page summaries of this information, just click on the hyperlinks below:

- [How Your Memory Works](#)
- [Seven Study Attitudes Worth Cultivating](#)
- [Seven Ways to Improve Your Study Environment](#)
- [Seven Steps to Follow Every Evening](#)
- [Four Ways to Study Mathematics](#)

## Four Ways to Study Mathematics

Each time you sit down to study mathematics, you will be studying mathematics in one of four ways:

### Method #1 ~ Studying Things You Need to KNOW

The first involves information that requires memorising. This can consist of names for objects, definitions, spellings, quotations, formulae, etc. It is a good idea to keep a book or folder for this purpose and devote a page to each topic/concept. I will try to provide summary sheets for you on the [CrystalClearMaths](http://www.CrystalClearMaths.com) website, but it is better if you create your own as you will be more likely to remember your own summary.

Choose a topic, select a blank page or sheet of paper and open your text(s) to the relevant chapter(s). Now write down all the things that you need to know that you don't already know fully. This does not mean that you simply recognise the material; it means that you are confident that you could recall that information perfectly at any time that you might be asked for it. If there is any doubt in your mind, write it down. Concentrate also on correct spelling as it is necessary for clear communication.

Once you have scoured the chapter(s) for all the things you need to know, put your text(s) aside and rewrite the summary page neatly and in an order that makes sense to you. Take time with this and plan it carefully.

Now, while it is fresh in your mind, put the summary sheet somewhere out of sight and, taking a blank page, attempt to rewrite the entire summary from memory. You are most unlikely to get it fully correct on your first attempt. You may only recall a small portion of it, if it is very complicated (for example, having to learn names for all the bones in the human body, if you are studying medicine). Compare your efforts with your summary sheet.

At this stage, you have two options, depending upon how much time you have. If your time is limited, because of assignments and homework load, put the summary away and make a note in your diary to deal with this within the next couple of days. If you have time, keep writing your summary from memory and comparing it with the master summary that you creat-

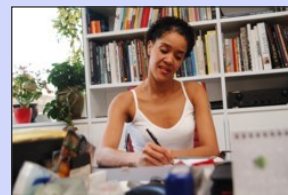
ed. Stop when you feel that you have improved sufficiently and make a diary entry to check it again within a few days.

Your goal is two-fold: to learn the summary sheet, and to ensure that it becomes part of your long term memory. This is why you use your diary, to keep practicing for a few minutes at a time until you can recall the summary after weeks or months of no revision. For some sheets, such as learning all the names of parts of a circle, 2-4 minutes per day will probably be sufficient. Others, involving definitions (or quotations, if you are requiring them for some other subject), may require 15 minutes each time until you know them well.

In time, you may even make summaries of your summary sheets, eliminating things that you now know without any doubt, so that your 'summaries' will get smaller and smaller. After a few months, you can return to the original summary to see if you remember all that it contains.

You can enhance your memorising skills by:

- speaking out aloud as you write as this ensures that you use quite a few senses/actions (touch, voice, hearing, sight) and this can help you learn much more rapidly than if you simply read your notes.
- moving when you learn (some people benefit from walking around the room or even from walking/running around the neighbourhood while they listen to or recall summaries).
- using your sense of rhythm by putting some of your notes in poetic or song form and learning the poem/song.
- using mnemonics (memory aids) such as acronyms. Acronyms are formed when the first letter of each word is used to form a new word. For example, when multiplying two binomial expressions, many students use the acronym FOIL ... First, Outside, Inside, Last. Many learn the colours of the rainbow as though they form a man's name, ROY G BIV ... Red, Orange, Yellow, Green, Blue, Indigo, Violet. You might be able to make up clever acronyms to recall the main points concerning an event in history or a musical piece etc. Acronyms can



**Take time to summarise the information that you need to know. Hand-written summaries use more senses and are remembered better.**

*Your goal is two-fold: to learn the summary sheet, and to ensure that it becomes part of your long term memory.*

**First  
Outside  
Inside  
Last**

**Acronyms can be very helpful for remembering lists or the order in which to do things.**



**We learn (study) all our lives. To learn a skill well requires practice.**

*The great samurai swordsman, Myamoto*

*Musashi wrote in his Book of Five Rings, "You can only fight the way you practise," so practise for excellence.*



**Whether you are learning a skill or gaining knowledge, listen carefully to your instructor and 'get it right' from the start.**

be extremely useful even in mathematics.

- using other mnemonics such as the number of letters in a word to recall the value of  $\pi$  to a given number of decimal places. To recall  $\pi$  to seven decimal places is actually quite easy ... just learn the sentence, "How I wish I could calculate pi." The number of letters in each word gives 3.141592! Google "mnemonic for pi" and read some of the interesting attempts that have been made.

There are many good videos on [YouTube](#) teaching clever ways to use memory keys/pegs to remember lists. Experiment with memory techniques and search for them on the Internet. You may find some very funny and clever ones that will help you remember those difficult-to-recall facts.

### **Method #2 ~ Studying Things That You Need To Be Able To DO**

This time our discussion will focus on 'skills.' These are things you need to be able to DO. The only proper way to learn skills is to practice them!

Mathematics is possibly the most skill-based academic subject that you will learn (learning drama, dance, a sport or learning a musical instrument are also very skill-based). This means that a very large proportion of your study time will be spent DOING mathematics.

The best method that I can recommend is to ensure that you do the set homework each day and continue doing questions until you are confident that you fully understand the work. This means that you need to obtain another book of exercises or download exercises from the Internet (I will be endeavouring to provide this kind of resource for free on the [CrystalClearMaths](#) website). Always set your work out neatly and methodically. Train yourself to do things well, not messily. The great French chef, Fernand Point, wrote in his classic work, *Ma Gastronomie* (1969), *le succès est une somme de petites choses mises au point* (Success is the sum of little things done well). This is the attitude that will bring you success in mathematics. Also, the great samurai swordsman, Myamoto Musashi wrote in his *Book of Five Rings*, "You can only fight the way you practise," so practise for excellence.

When you have finished the questions and are confident, use your diary again. Ask yourself how long before you might be at

risk of forgetting how to solve this kind of problem and make a note in your diary to practise some on that date. If you could not understand the work, ask your parent or teacher to show you more clearly or search the Internet for videos (I will be endeavouring to provide this kind of resource for free on [YouTube](#) as well). It is important that you do not let this opportunity pass. Never finish a topic or a chapter without being confident that you can do the work required for/in it. This requires effort at the time, but it makes study so much easier later on!

It is also a good idea, if you want to really excel in mathematics, to seek out difficult and challenging questions and work at them until you understand and can solve them. Many school text books have a collection of more challenging questions at the end of each chapter. It is a good idea to try to solve all of them if you really want to master the skills for that topic (again, on this website, I intend trying to provide a range of challenging and extension questions for most topics/skills).

If you are very determined to become a good mathematician, there are two attitudes that I can recommend:

- Determine that you will never leave a single solution incomplete. That is, make sure that you succeed in answering every question that you encounter in your school course. This means that, if you get an answer 'wrong,' you keep working at it until you get it 'right.' Sometimes it may mean spending up to a half hour or so (in extreme cases up to an hour or three) on a single question! Never give up! In some cases you may have to have your teacher check that the answer in the text is correct (in some schools that used poorer text books I used to encourage my students to have races to find the most errors!).
- Determine that you will be able to answer every single question in your text book. This does not mean that you have to do them all, but it does mean that you will have examined each one and been satisfied that you can do it confidently and quickly.

Finally, knowing the skill and remembering the skill are still not quite sufficient. There remains the matter of speed. It does not help if you know the work but cannot finish an examination in time. I recommend that, as part of your homework as well as your regular revision, you race using a stop

watch or timer. For example, if you have forty equations to solve, time yourself on the first five and then work out how long the remaining 35 should take you ... and then try to beat that time in the one sitting. Generally, you will get faster and beat your time, so you will gain focus and encouragement from doing this. You should always time yourself when completing past exam papers.

Part of building speed is to practise solving lots of smaller problems in your head at great speed. For example, you might try to solve a page of algebraic equations in your head (and someone else can check your answers as you call them out). Progress from two step to three step to four step equations and beyond. This is an excellent use of time. In two or three minutes you can solve more problems orally than you could in twenty minutes or more on paper and your mind will be better for it! Be sure to include oral drills like these in your study times. I will be attempting to create spreadsheets and post them on this [CystalClearMaths](http://www.crystalclearmaths.com) website so that you can download them and use them for this kind of drill work.

Your overall aim is to build long term recall of a skill, and be able to use it confidently and with great speed. In short, when studying a mathematical skill, you need to move through three phases: understanding, speed and accuracy, and long term recall. Most students only do sufficient work to accomplish the first goal. They fail to build speed and accuracy and do insufficient to recall the work properly when it is needed months later. If you accomplish all three, you should excel!

### Method #3 ~ Learning How to Avoid Making Careless Errors

Studying to avoid careless errors is a particularly good thing to do after exams. When I was teaching I found that the average student lost 3-15% of their exam marks due to carelessness (the average was around 8%)! This was painful for my students because it was material that they had studied and already knew.

If someone offered to raise your marks 3-15% for free you would have to be impressed. Of course this would make a big difference to your results!

But, how can you avoid carelessness? The answer is simple. How do scientists avoid accidents in a laboratory? No one sets out

to have an accident any more than they set out to lose 8% of their exam marks due to carelessness. To minimise the number of laboratory accidents, scientists set rules and teach each other to **make a habit** of behaving in particular ways. For example, they learn NEVER to look down the neck of a test tube and ALWAYS to wear protective goggles. The incidence of eye damage in laboratories where these rules have been practised has been almost eliminated. You need to do the equivalent thing for each of your errors in your tests and workbooks.

When your mathematics exam paper is returned, don't throw it away like a lot of students do. That same night, sit down with it and work your way through the exam paper looking for questions that you got wrong because of carelessness – and try to work out why it was that you made the mistake. Was it that you misread the question, or did you round a number off incorrectly, or did you do something wrong with a minus sign? There are many, many possibilities.

When you have identified why you made each careless mistake, try to think of a habit that you could develop or improve upon that would help you avoid each of those errors. Some of the more common and simple solutions are these:

- drawing bigger diagrams, using colour, or placing diagrams elsewhere on the page (one of my student's results increased dramatically when he drew diagrams 8-9 cm square instead of about 4 cm square);
- changing the order in which you do things (it may be that you are not completing one action before beginning another);
- producing an extra line or more of working and not trying to complete more than one step at a time in your head (a very common error made by students who wish to rush or 'get things over and done with' is to write too little and thereby make errors ... a common problem among students with talent);
- setting your work out with headings (this simple act can sometimes make each step that much clearer);
- writing the appropriate formula out BEFORE substituting values into it;
- developing a habit of re-reading each question when you have finished answering it (to make sure that you have answered it completely); and



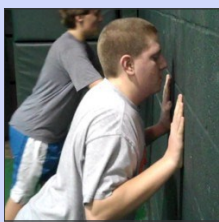
**Mental speed drills can help you achieve mastery over skills in minutes rather**

*Your overall aim is to ... move through three phases: understanding, speed and accuracy, and long term recall.*



**To avoid costly mistakes, we use protective gear and good equipment and learn proper working procedures.**

**Developing good habits in mathematics also results in fewer costly errors.**



**You may not move the wall but this can still be great exercise!**

**Even if you have not solved a problem, the effort you have made has not been wasted!**

*The developing of mathematical skills should not be an end in itself ... look for opportunities to think mathematically and be challenged with more difficult and unusual problems.*



**Some habits are not so helpful or healthy for you.**

**Try to cultivate good habits in your work and study.**

- solving the entire problem in one big equation instead of lots of little ones (this can be very useful when finding composite areas, for example).

There are other possibilities as well, but this is a good sample for you to start with.

What we're trying to do, of course, is to establish **habits**.

One lot of habits is concerned with how you set your work out on a page. I can't emphasise enough how important this is, not only to think a question through and solve it properly, but to practise clear setting out until it is second nature.

Another group of habits relates to how you go about answering your questions. It's good to establish a procedure or a routine that you follow faithfully in your homework and your study and make such a habit that it won't fail you in exam time. For example, you might read a question, draw the diagram, reread the question to check that your diagram has been constructed properly, write your equation, reread the question or recheck the diagram to make sure that you copied the equation correctly or that you constructed it correctly based on the information given. When you've finished solving the problem, copy the entire answer from the calculator screen, then reread the question to find how you should truncate your answer and what kind of units are required. This is a good habit to develop. Now, you might refine it in various ways of your own, and you need to find a system that works for you, reduces your error rate, and one that you can live with and practise faithfully. You need to practise that skill until it is so engrained, so deeply engrained, that it won't fail you when you come to solve mathematical problems or to answer questions in your mathematics exams.

Explore ideas that will work for you. Be creative. Follow your teacher/tutor's instructions carefully and watch how they solve problems. Think of habits that you can develop that will help you avoid careless errors AND THEN PRACTISE THEM.

#### **Method #4 ~ Spend Time Working on Challenging Exercises**

The first three methods (how to learn facts, learn skills and avoid making careless errors) will form the basis of all your mathematics study. They are roughly equivalent to learning spelling, grammar and syntax in

English — these are all very essential for good communication, but they only provide the basis upon which all literature is developed (poetry, songs, short stories, novels, sagas, etc.). Similarly, the developing of mathematical skills should not be an end in itself. Rather, you should look for opportunities to think mathematically and be challenged with more difficult and unusual problems.

Some textbooks do contain challenge questions or challenge exercises but, if they do exist, they often take up only a very tiny part of the book. If you want to really develop your mathematical muscle, you need to seek out questions from other sources. I can recommend asking your teacher whether the school has resources that you might borrow or use or have access to. Subscribe to a students' mathematics journal if there is one in your state or your country. Ever since I was in school, a long time ago, I've been subscribing to a mathematics journal called "[Parabola](#)" which is specifically written for high school students. I still enjoy reading the articles and find them stimulating.

Seek out challenging competitions. Ask your teacher whether there are mathematics competitions that you can enter.

Try to find books of puzzles and challenges in your local library or your school library. And be prepared to spend hours on single questions. There are lots of unsolved problems in mathematics, but it doesn't stop mathematicians from working hard at them. This is good experience!

Be prepared to spend hours solving a single problem. Some puzzles you may never solve (there are many unsolved problems in mathematics), but your time will not have been wasted! I explain to my students that, if they were to push against a wall for fifteen minutes each day, they would get very fit and strong even though the wall would not move. If you spend two hours trying to solve a problem in geometry, you will have been trying all kinds of strategies, recalling many different theorems, properties and rules, exploring all kinds of constructions — and it is all of great benefit to you. The mere fact that you failed to answer a question or solve a problem doesn't mean that your time's been wasted. In fact, it's some of the best study that you will ever do!

Of course, if you do manage to solve the problem or, better still, discover a principle that very few others might know, the feel-

ing of achievement and satisfaction is immense! I still remember discovering a theorem in geometry when I was about 14. The feeling was wonderful; and I remember it all these years later. It's this kind of feeling that drives mathematicians to further exploration and further study and further research. It is this feeling that few school students experience with intensity in mathematics but it drives good mathematicians to further exploration.

So, as the fourth part of your study system, choose to tackle difficult problems occasionally and go exploring! The rewards are truly immense and satisfying.

### Choose Variety

Now that you know the four different ways of studying mathematics, you can use this knowledge to create variety and interest for yourself when you're studying. For example, if you don't feel like doing a lot of exercises on a particular night (Method #2), you might decide to create or revise a summary instead (Method #1), or look over one of your past papers to explore different ways of doing things (Method #3), or even snuggle down with a problem that you have found or thought up for yourself (Method #4).

## Seven Aspects of a Balanced Lifestyle

### Be Kind to Yourself!

For good health and a good mind you need:

1. **Exercise.** Almost without exception, top students exercise and observe that exercise makes their mind function more clearly and increases their stamina when it comes to study. Also, fit people need less sleep and have better quality sleep. They even get sick less often and their illness is less severe. Consider incorporating some exercise in your daily activities, such as walking or cycling to school or walking a pet. Some students like the flexibility of exercising alone. They will have exercises that they perform at home including running, skipping, punching a bag, playing with soccer balls or basketballs etc., or skateboarding. Many good students like regular team sports which means they get to socialise with friends at the same time. Endorphins from exercise will help reduce your stress levels and the

Keep this list handy (or print a one page summary from the hyperlink on page 16) and, if you are feeling a bit 'flat' or demotivated one evening, choose a study method that you might not have used for some time.

Let me encourage you to keep checking this website as well, or perhaps even [sign up for our newsletter](#). I've let you in on a little bit of a secret here. As I keep adding material, topic by topic to the mathematics sections of the website, I intend adding them in these four categories — that is, I'll be adding summaries (so you can learn **facts**), I'll be providing work sheets and drill sheets so that you can practise these **skills** over and over again (both on paper and in your head), I'll be providing videos that show you quite good methods and habits to develop so that you can reduce and perhaps even **avoid making careless mistakes**, and I'll also be providing **challenge questions and puzzles** for you, topic by topic, as well. So, please keep a lookout for them, and I trust that you will enjoy them and find value in them. Please be patient with me because it's an absolutely massive task but, if you have constructive suggestions to make, or perhaps wish to make a request, then please let me know via the [feedback](#) form on the website.

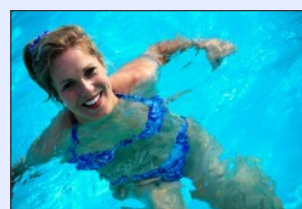
focus that many sports require will help you 'zone out' and not even think of anything school-related for a while. If you have a job, physical work can be good but stressful jobs and those taking up too many hours per week can be counter-productive.

2. **Eat good, healthful food.** Most top students benefit particularly from their parents providing regular and nourishing meals. Good food means a more active and alert brain ... essential for top performance. If this is a weakness in your environment, try to negotiate for less 'junk' food to be available at home, for a bowl to be stocked with fresh fruit and nuts (or keep your own near your study room). Your brain is an amazing organ. Your brain makes up only about 3% of your body mass but accounts for about 25% of your total body glucose utilisation (reference [here](#)). But refined sugars are [very bad for you](#) (see [this](#) also), and can even lead to [mental illness](#).



**Good food nourishes good minds!**

*The developing of mathematical skills should not be an end in itself ... look for opportunities to think mathematically and be challenged with more difficult and unusual problems.*



**Almost without exception, top students exercise.**



It is vital that you get enough sleep.

Sleep is always better than unproductive study.

*You brain only makes up about 3% of your body mass but it [can consume] around 30% [of your oxygen uptake] when you are involved in intensive study.  
Study is hard work!*



Spend adequate time with friends.

Learn to "talk each other's love language" (see right).

[es](#). So, learn to eat a low GI diet (glycaemic index — [see Wikipedia article](#) and a helpful list [here](#)) and you will benefit enormously. Also ensure that you remain well hydrated, i.e. drink sufficient water. Minimize your use of sugary drinks and those laced with caffeine and similar stimulants as these are not good for your body and overuse will have negative consequences.

3. **Breathe fresh air.** Your brain only makes up about 3% of your body mass but it consumes about 20% of your oxygen uptake when resting and this can increase to around 30% when you are involved in intensive study. Study is hard work! Ensure that you get adequate fresh air and I plead with you not to smoke. Apart from all the other chemicals involved, carbon monoxide combines with the haemoglobin in your blood to produce carboxyhaemoglobin, which is ineffective for delivering oxygen to bodily tissues ([see Wikipedia article](#)). Cigarette users inhale about four times the carbon monoxide found in car exhaust and about 5% of a pack-a-day smoker's red blood cells are carboxyhaemoglobin. If they smoke the pack within an eight hour period, this will rise to 7-15% ([see carbon monoxide article](#))! Please be kind to your body.

4. **Rest.** It is absolutely vital that you get enough sleep! Almost every quality student that I have asked has testified that this was a major factor in their success. Studies show that insufficient sleep can seriously affect your ability to recall and consolidate information and can also lead to increased appetite and obesity. It appears that consistently sleeping too much can lead to significant health problems as well. The same studies show that your optimum sleep time is a very individual thing, so experiment to find what amount leaves you feeling the most fresh and alert and plan to get to sleep at a regular time each night, especially during the school week. Try to take breaks at significant times during the week when you simply rest. Lie down ... on a couch, bed, floor, or in a park, and relax. Even ten minutes of this can make a big difference to your functioning during the day. It is also worth noting that many top students plan their week so that, on one particular night, they do no study at all. Finally, do not attempt to study when you are over-tired ... sleep is always preferable to unproductive study.

5. **Build your social relationships.** Sadly, not all parents/guardians are supportive, but most desire the 'best' for their children. Let your parents/guardians see that you are making an effort (the best students are self motivated) and then negotiate for their support. Spend adequate time with your friends (some students need more time with friends than others) but be disciplined enough to resist some of their requests/demands if they are less studious than you. YOU need to be in charge of your time management, not them. Something that I have found to be extremely important is an awareness of my 'love languages.' Gary Chapman has written a series of excellent books about five love languages that we all share ... words of affirmation, gift giving, quality time, physical touch, and acts of service. Visit [Gary Chapman's 'Five Love Languages' website](#) and complete the appropriate quiz to find which ones are dominant for you. Share the results with significant people in your life and they may decide to encourage you by giving you more of what really 'switches you on.' A bonus is that, in completing the quiz, you may gain insights into how to bless and encourage your peers, teachers and other members of your family. All this will help build harmonious relationships that will strengthen and support your study endeavours!
6. **Debrief.** Build relationships with a few friends, your parents/guardians, your teachers, and with people who are not connected with your schooling so that, when you are stressed, you will be able to find someone to chat with who will provide a good listening ear. Sometimes it will help if they are in the midst of it with you (such as a school friend or a trusted teacher or parent) but sometimes it will be good to chat with someone quite removed from the scene who can give you sympathy and perspective (such as a grandparent, a local sports coach, a local minister or counsellor, et al.). It helps to have supportive people in your life who can give you comfort and sympathy when you need it and encouragement and perspective when you need it.
7. **Create variety.** Whether you manage your study in a less disciplined way or whether you organise almost everything, you need to have a very varied life. Try also to maintain a hobby (or two) if you can.

## Fourteen Study-Enhancing Lifestyle Choices

1. **Build and cultivate friendships with other achievers.** By all means maintain friendships with those who do not wish to study, but be aware that they will apply all kinds of social pressure to drag you away from your studies. They will not understand your drive and will try to undermine it so that they will feel less guilty and, on the positive side, simply because they enjoy your company. Make sure that you have a very good group of like-minded friends who will not ridicule you. You can share meals, parties, social activities, study time and sports with them. It is easier if they are in your classes at school and easier still if your classes are 'streamed' so that students of similar ability are grouped together. It is unfortunate that a lot of our social groups value high achievers in sport, drama, dance and music but prefer to pour scorn on academic achievers. It is important for your health that you find your niche and support group where (your) academic achievement is valued.

2. **Build a good relationship with your parents (and others).** A good relationship with supportive parents can make a huge difference to the quality of your study and life in general. I am aware that this document may have a wide audience and that, sadly, not all parents are as devoted to their children as they could be. None-the-less, without being responsible for your household, endeavour to build a relationship with your parents. Read about the [Five Love Languages](#) and determine which ones apply to your parents. Endeavour to do things for or with them that are meaningful for them. Ensure that you do some chores around the house without being asked (extra ones ... as simple as sweeping a hallway). With most people, as you attend to their needs, they will begin looking out for your needs more. I encourage you to develop a good relationship with some other adult(s) in your life as well ... grandparents or other relatives, coaches, teachers, neighbours ... but do be careful as not all adults are necessarily trustworthy. If you follow these suggestions, this could be the beginning of a more open relationship with the adults in your life. The benefits for you should be a less stressful environment and more

sympathy for your study (and other) needs.

3. **Don't treat your teachers as enemies** ... no matter what your age. There are unfortunate exceptions, but the majority of teachers are very dedicated to helping you and put up with a lot of abuse, hard work and long hours for their students. They (and even the exceptions) can respond well to an act of kindness or an encouraging word or active involvement in a lesson. Try treating your teacher as a real, live human being (and encourage your parents to do the same) and see what happens. You may be wonderfully amazed. A teacher working hard in cooperation with you can make sure that your hours of hard study are not wasted. Win a friend and ally, don't make enemies. You don't have to be in your final year of school to do this and it does not mean that you have to become a 'teacher's pet.' It simply means that you are being more adult/mature in how you relate to others.

4. If you have good friends of similar ability doing the same course(s), you might **engage in a bit of friendly rivalry**. Most students thrive on this and it can make the study more challenging and fun. Study with them if you can, but try to beat them (in a friendly way) in the exams. A very large proportion of top students engage in this kind of friendly rivalry.

5. Take time to **help others** and explain work to them. Not only is it good for them, but you will learn a great deal by having to explain the work and answer their questions. We are all 'social learners' to a degree (some much more so than others), and it is good to spend time discussing studies with others.

6. **Control your use of social media.** Turn devices off during serious study times and then enjoy them without guilt at other times. Many good students use Facebook to collaborate with friends at night and to get help and advice while doing homework. This can be a very good thing and should be encouraged. Try to be disciplined with its use, though, and do your serious study in a different room or where you won't be tempted to log on.

7. In classes at school ensure that you **listen carefully, take notes and ask ques-**



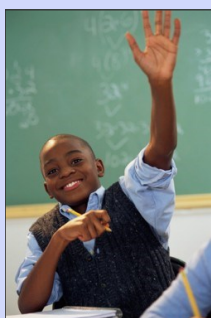
Try to build a good relationship with some significant adults in your life.

*Try treating your teacher as a real, live human being and see what happens ...*

*A teacher working hard in cooperation with you can make sure that your hours of hard study are not wasted.*



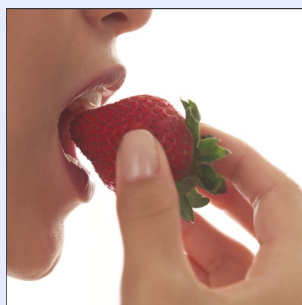
Take time to help others and explain work to them. Not only is it good for them, but you will learn a great deal by having to explain the work and answer their questions.



Asking questions in class saves a huge amount of time and effort at home.

*If you are feeling totally wiped out, do not study!*

*Your time will be far better spent if you got some good exercise, drank sufficient water and had a good sleep.*



Eat healthful food.

Fresh food should give you a surge of energy and alertness whereas junk foods tend to dull your ability to concentrate.

**tions.** If you are in doubt about anything, don't assume that you will be able to work it out for yourself later, ask your teacher for help. Be prepared to ask whatever questions that are relevant to the topic and that you need help with. Encourage your friends to do the same. Make it a class culture. A question quickly answered during a lesson can save literally hours of misguided work at home! Good students ask questions and listen carefully because it saves a lot of TIME.

8. Search the Internet and books and ask your teachers about **learning styles**. Experiment and develop the ones that help you the most. Draw posters, make flash cards, place stickers on things, use highlighters, etc. Find what works for you and make sure that it does so!
9. **Never skip breakfast.** This is a most important meal for the day. Without it, your ability to concentrate for the first half of the day is severely compromised. In fact, you should snack every few hours. We at *Crystal Clear Mathematics* are not dieticians, so ask others and read about good eating habits (you probably learn about them at school). Put good eating principles into practice. Don't delay. You will be better off for the change.

10. **Reduce your access to junk foods.** Study can significantly affect your eating habits and the temptation will be to snack on junk foods. For some people, the stress of serious study causes them to lose their appetite. For others, their appetite may increase dramatically and they can battle with their weight. If this describes you, then you need to ensure that you sleep sufficiently, that you avoid refined sugars and junk foods (negotiate not to have them in the house if you can ... and ask for salads, vegetables and fruit instead) and that you drink sufficient water. All these things will affect your appetite (and weight, and ability to concentrate).

11. **Stay hydrated.** If you want to be able to concentrate on your work it is vital

that you drink an adequate amount of water.

12. **Avoid drugs and stimulants** including all illegal drugs and tobacco. They are detrimental to your study and academic performance and the vast majority of top students avoid them totally. You might choose to have small amounts of alcohol and caffeine (coffee and tea) but be very careful that you do not abuse your body with them. My experience is, and surveys seem to confirm, that the very best students use far less of these than students who do not care about their academic performance. Again, many of these very good students avoid them altogether. They know that they need to function well the next day, and the next, and the next ...
13. **If you are feeling totally wiped out, do not study!** It will be a waste of time. Your time will be far better spent if you got some good exercise, drank sufficient water and had a good sleep.
14. When offered subject choices at school, **choose subjects that you like.** If you have a particular career path in mind, find out what subjects are required for entry into that career. If you really hate some of those subjects you may need to rethink your choices. Generally, it is exceedingly difficult to study a subject that you really dislike. You will work much harder, and better, at subjects that you are more fond of. Do not be influenced too much by the teacher that you might get (they can change suddenly) or the marks that you might think you will get. Follow your passions. Having said this, I must caution you to ask a lot of others for advice for your particular situation and not make big decisions based purely on this website's suggestions. People closer to you will be in a far better position to give good advice and you should weigh it all up very carefully!

If you have put into practice the suggestions made to this point, you should now have a good study system in place, know what you need to do each time you sit down to study, and have taken control of your lifestyle as much as you are able.

But what happens when you need to rely on all your study?

What should you do near and during your exams?

This is what we now need to discuss!

## Seven Ways to Prepare for Your Exams

### Look After Your Body and Mind!

After you've studied hard in the days leading up to your exams, the single most important factor that's going to influence your performance is STRESS — physical, emotional and mental stress!

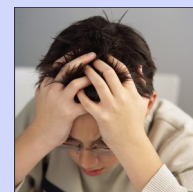
If I asked you to multiply 8 by 8 or 9 by 15, it might take you a few seconds. But, if you were threatened with physical or emotional pain while making those same calculations, your mind would tend to freeze up. This is precisely what tends to happen to many students during their exams — they 'blank out.' Now, it may be that they're simply not well prepared but, for many students, it is because of stress. Stress causes you to lose marks that you have studied hard for and that you deserve.

Your battle with stress is perhaps the biggest battle that you're going to have around exam time ... so here's my list of seven practical things that you can do to reduce your stress levels and to give yourself the best possible chance to perform well during your exams.

1. **Eat, drink, exercise, sleep and rest well** in the days leading to your exam(s). You need your body and mind in top working order.
2. **Don't stay up late cramming the night before your exam!** Studies comparing students who cram with those who get a good night's sleep show no detectable difference between their results. It is thought that any gains made and new things learned during the night were offset by poor concentration, poor recall and careless errors on the day. It is far more preferable that you get a good night's sleep and arrive at your exam fresh and alert.
3. **Prepare all necessary things the day before.** Make sure that everything you will need for your exam is in working order and packed before you go to sleep. This will considerably reduce the stress of the morning and you are far less likely to forget anything. Make sure that you have a spare battery for your calculator (or a spare calculator) and that you have an adequate number of pens/pencils in case any break or run out of ink.
4. **Eat a good breakfast**, taking care to avoid sugars and foods with a high GI

(glycaemic index — [see Wikipedia](#) and a helpful list [here](#)). Many students eat a sugary breakfast and it causes an almost immediate surge in their blood sugar level but it then plummets about an hour and a half afterwards, just at the time they're sitting for their morning exam. It is far better for you to eat slow energy release foods. It may also help to have a small snack half an hour before the exam starts and, if you are allowed to take foodstuffs into the exam room, to take a handful of nuts or fruit or some other low GI food to snack on about halfway through the exam.

5. **Do not talk with people (including friends) before the exam.** Just like athletes about to race in the Olympic Games, focus on your task at hand and/or spend your time relaxing. Too often, students gather outside the exam room and stress each other to the point of agitation, comparing notes and things that they have studied. Having listened to many of these public conversations I have observed that, often, students share incorrect information which stresses and confuses others even more. It is best to avoid such gatherings. Arrive at the venue in plenty of time but stay at a distance and only walk over to the examination room when the doors are opened.
6. **Go straight to your desk** and don't make eye contact with other students if possible. Maintain your focus. This may sound rude, but it is in your best interests. Seeing friends rolling their eyes or distracting you, or viewing others under considerable stress will do nothing to help you. I recommend sitting with your eyes shut for a while (relaxing), or looking at your desk or the notice board at the front of the room (if there is one).
7. **Don't stay and debrief after the exam**, or hold a 'post mortem' with friends, especially if you have further exams to complete. There is nothing you can do or say that will change the results for your last exam. These gatherings usually increase your stress levels rather than decrease them. By all means, socialise and enjoy your friends' company after your last exam is over — but, if you want to keep your stress levels minimised during the exam period it's wise to avoid these gatherings.



**Don't stay up late, cramming before exams.**

**Studies comparing students who cram with those who get a good night's sleep show no detectable difference between their results.**

*Your battle with stress is perhaps the biggest one you will have around exam time ... so try to do nothing to increase it.*



**Eating a good breakfast is vital, and a snack half an hour before the exam can help keep your brain alert.**



If you have studied well, your major battle during exam time is stress.

Stress reduces your capacity to earn the marks that you deserve.

*It would be a very pleasant thing indeed to have your marks jump by around 8% with essentially no extra study, wouldn't it?*

*Stress is a major problem.*



Take time before your exam to rest, relax, meditate and/or pray.

## Seven Hints for Sitting for a Mathematics Exam

### Do Everything to Minimise Stress and Maximise Results!

The average mathematics student loses about 3-15% of their examination marks due to 'carelessness.' The average is around 8%. I have verified these figures with a number of my classes. This represents quite a large loss of marks.

Now, please understand that, if you are losing these kinds of marks due to carelessness, this is work that you already 'know.' These marks are ones that you have studied for and 'deserve' but they are lost because of a combination of **poor habits** and **stress**. I have written elsewhere about how to study to develop good work habits (How to Study Mathematics, Method #3), and I have explained in the last section (Seven Ways to Prepare for Your Exams) how to minimise stress during your exam period. This is all general preparation for any exam.

When I observe students' performances during mathematics exams in particular, I have noticed that many students lose marks because of **poor time management** or **poor planning** during their mathematics exams. If you suffer from similar problems, then here are seven practical things that you can do during your exams to help you earn the best results that you can:

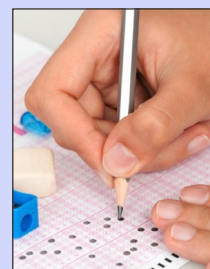
1. First, **spend time calming yourself and relaxing before the exam even begins**. Close your eyes, stretch gently, breathe deeply, and rest. There are many good websites with advice concerning relaxation techniques. Read them and adopt the ones that work for you.
2. Second, **calculate how long you have to earn each mark**. Normally, you'll be told the length of an exam and how many marks are allocated before the day that you sit for it. But, even if a test is given to you suddenly, it's worth spending a few moments calculating this figure. Knowing this information can help you avoid spending too long on any particular question. In my country most major exams seem to be based on one-and-a-half minutes per mark. This means that, if you encounter a three mark question, you should allocate no more than four-and-a-half minutes to answering the question.
3. If you are given **reading time** before the exam starts, use it to **scan through the exam paper** looking for questions that you can answer confidently ... simple ones or questions on topics that you have studied very thoroughly and find quite easy to do. Don't worry too much about questions that look difficult. They will be there whether or not you worry and you will, in fact, be able to solve them more easily later in the exam. Your task at this stage is purely to find the 'easy' questions and form a plan in your mind as to what order you will complete them in once you are allowed to write.
4. **Spend at least the first 20-30 minutes answering those questions that you have already identified as 'easy.'** This is the period where your stress levels can reduce dramatically, so it should be a calming time. Do not even attempt to answer a question that is confusing or difficult for you. If you start working on a question that proves to be more difficult than you first thought, abandon it immediately and make a note to return to it later. Your primary aim is to ensure that you can work swiftly and efficiently during these 20-30 minutes with as little stress as possible. This will allow two very important things to happen. First, the stress response in your body and the stress hormones in your blood stream will diminish markedly. This will allow you to think more clearly for the remainder of the exam (when you can start dealing with the more difficult questions, so you will actually answer them more successfully anyway). Second, you should find that you have raced through a significant amount of the exam paper with a very high success rate. For example, after thirty minutes of a one hour exam, you may find that you have already answered 60-65% of the questions and earned 50-60%. This would give you the remaining half hour to finish the remaining 35-40% of the exam and earn whatever extra marks you can. It's a very comforting feeling to know that you've earned 50% by half time. And this allows you to relax even more and, potentially, perform better in the second half of your paper.
5. **Do not get unduly bogged down** on any particular questions. If you've al-

ready invested more time on a question than the marks would warrant, it's time to leave it and go to other questions and return later on. Make a code to keep track of the questions that you leave and return to. Some students will circle the question number when they have to leave and shade it in when they answer the question successfully later on. It is senseless spending time on a difficult question when you could be earning marks on other questions elsewhere in the paper.

6. If there is a **multiple choice section** in your exam, seriously consider completing it **last** ... at least, after you have earned all the marks you can in the longer part of the paper. The great advantage of multiple choice questions is that you always stand a reasonable chance of guessing the answers if you are running out of time. Even when working through the multiple choice section, however, watch the clock and allocate only the proper amount of time per mark (most multiple choice questions are one mark each). Again, if you get stuck on a question, mark it as incomplete and come back to it later (if you have been able to confidently eliminate one or two of the options, cross them out as this will mean less work for you when you re-

turn). Make absolutely certain that you have attempted every question first and earned all the 'easy' marks that you can possibly get **BEFORE** you go back to work on the more difficult ones. It's also a good idea to ask for some spare paper and to set your work out as you answer multiple choice questions. I have seen many, many students lose lots of marks carelessly because they've tried to answer multiple choice questions in their heads. It's also a good idea to answer multiple choice questions, if you can, before you get too distracted by the options given.

7. Finally, if you finish with time to spare, spend a short time with your eyes closed, breathing deeply and stretching your arms and legs in front of you (never up or out in an exam room) and resting/relaxing a little before going back and **checking your results**. When you re-view questions, it's a good idea to try to answer them in a different way to see if you get the same answer, or to review your answers to see whether they fully satisfy the conditions of the question. There is no merit in walking out of an exam room early only to realise that there were questions that you could have answered better! Checking is a very good policy.



**It is a good idea to leave multiple choice questions until last. At least you stand a reasonable chance of guessing the answers.**

*Spend at least the first 20-30 minutes of an exam solving the easiest questions.*

*This is the period when you can work to reduce your stress levels dramatically.*

## How to Sit for an Essay-Based Exam

### Plan the Essays All at Once!

This topic, in a sense, does not belong in a document that is designed to teach you how to do well in mathematics. I am adding it for two reasons. The first is that I believe that I have something useful to share with you about sitting essay-based exams. The second is that, if you perform better in other exams as well, then your confidence levels will rise generally, your stress levels during your 'exam week' may be lower, and you may even find that you have to study some subjects a little less intensely (and that can mean more time to study your mathematics).

This suggestion is only of use if you have a number of essays to write. The more essays, the more useful this will be. I have had to sit for quite a few exams that required five essays to be written and found that this system worked very well indeed.

1. If you have reading time for your exam, use it to choose the questions that you are going to answer.
2. The moment that your exam starts, select a sheet of paper for each question and write the question number at the top of each page. If you have to write your essays in booklets, ask for spare paper so that you can do this.
3. Now, read each question in turn and, on the appropriate sheet of paper, spend a few minutes writing down absolutely every fact and quotation and bit of information you can think of that's relevant to that question.
4. When you have finished this process for each of your essays, five or ten or even fifteen minutes of your exam time might have elapsed during which time the students around you have been working on their first essay — but, you have four big advantages! The first is that you no



**Plan all your essays at the beginning of the exam.**

**This reduces your stress and allows your subconscious mind to think of more points to make for later essays.**



**Make sure that you have all your equipment packed and in working order during the night before an exam.**

*As you answer each question, first reread the question, and then spend another minute reading over the notes on the summary page ... then write the essay swiftly and with confidence.*



**Include your study as part of a balanced lifestyle.**

**Enjoy your time with friends but remember that academic success brings its own rewards as well. It is definitely nice having both!**

longer have to struggle to remember anything for any of your essays. Your notes are all on paper and this means that your mind is free to concentrate on each essay as you write it. The second is that your subconscious mind is now fully acquainted with each essay and, as we'll see, that's a very important fact. Third, your stress levels are now vastly reduced and this means that you can think more clearly. And fourth, even if you had to suddenly leave the exam room, you still have the potential to earn good marks on all of your essays simply on the basis of the notes that you have written.

5. Now place your papers in the order in which you're going to write your essays, from the easiest one on the top to the one with which you're least confident on the bottom.
6. As you answer each question, first reread the question, and then spend another minute reading over the notes on the summary page, circling and drawing arrows and numbering sections to plan out the order in which you are going to make your points. Then write the essay swiftly and with confidence.

7. The reason this system works so well is that, while you're writing an essay, your subconscious mind will be popping up ideas relevant for the remaining essays! Simply pause your writing, find the relevant summary sheet and jot down the note that came to mind, and then resume your essay. I have sometimes found, by the time I've got to my last essay sheet, that I've had three to five times as many points as I had written down at the beginning of my exam time! This system has worked well for me and I commend it to you. I have found that my stress levels were vastly reduced after having jotted down all the notes for all the essays. Sometimes I have earned extremely good results on the later essays even though I began the exam fearing that I would not have sufficient notes to even get 50% for those questions.

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## A Personal Confession

### Ten beliefs or attitudes that influence my approach to study and learning mathematics

To this point I have shared, quite comprehensively, how to go about studying. In this last section I want to share something of a confession with you. This confession takes the form of ten observations that I've made (or ten attitudes that I have) towards study and towards students who study.

1. The first is that **there's not much point going to tutoring or going to classes if you're not going to bother learning** the material. I've encountered students in my classes, on occasions, who've spent thirteen years in school and still can't add or multiply properly, let alone understand percentages, decimals, algebra, anything else about mathematics — and you truly wonder where the thirteen years have gone. It just seems to me that, if you're not going to bother to learn, it is such an awful waste of time.

2. My second belief is that **not everyone needs to become a mathematician** — understanding things like algebra and trigonometry — any more than everyone should be forced to become a musician or an artist. We all have individual talents in life, and some have a passion for mathematical thinking.
3. My third belief is that **it's important that everyone have some understanding of mathematics**, particularly in our technological world. For example, in the average office, there's not a single item that has not been designed or manufactured or assembled or constructed or analysed without using mathematics — from the shape and size of the papers to the design of equipment (including computers, calculators and mobile/cell phones) to the chemical composition of the pigments in the paint on the walls. It's good to have some awareness of these things, even if one doesn't make a life career of

them. I believe this in much the same way that I believe that everyone should understand the language that they speak, and be able to read and write in that language.

4. My fourth belief is that **there is a beauty in the doing of mathematics**, whether or not you use the technologies available to you. Some like to use the 'old fashioned' method of using a pencil and paper and sheer mind-power — which I rather like — whereas others like to use technologies like calculators and graphics calculators and spreadsheets and computers to analyse problems in a different way — and I've used those methods as well. Friends of mine who are artists may enjoy painting and sketching but, at other times, pick up a camera and consider it a creative act to modify the image on a computer. Friends of mine who play musical instruments might enjoy playing an instrument in its own right but, at other times, might like to play around with the sound on a synthesiser. All these are valid ways of expressing your creativity in a variety of fields, and I don't see that it's any different with mathematics.
5. My fifth belief has to do with **motivation which is absolutely vital for good study** — and I've dealt with it in two or three of the precious sections (lifestyle) — always in a fairly general or, if you like, a secular way. But not everyone believes in that way. Not everyone believes, for example, that we came about by spontaneous generation or, if you like, that hydrogen turned into people, or that the universe is piteously indifferent and meaningless. If you believe there's a purpose to your life or that you're here with given gifts, then **perhaps your purpose and your motivation in study is to develop your gifts**.
6. My sixth belief is that **it is ok and it's right to encourage some students to excel in mathematics**. It's not just a 'nerdy' thing to do, any more than it is for students who want to excel in music or swimming or history or literature or any other field of endeavour. I get very disappointed in parents, for example, who disparage young people who spend a lot of time on mathematics — or teachers or parents who encourage students to spend a lot of time on their mathematics — when these same parents will take their own children to spend hours and

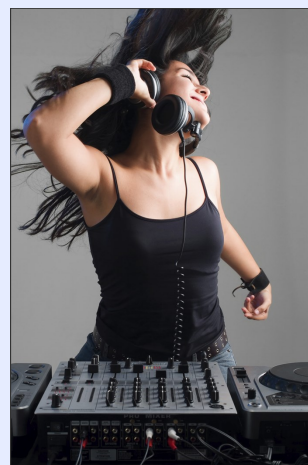
hours and hours playing soccer or hours playing football, or hours swimming up and down a pool to develop their swimming abilities, or hours practising at piano or violin or whatever other musical instrument. I think it's terribly unfair and it's terribly unbalanced, and I think it's right and proper that young people, if that's their passion and their natural ability, be given opportunity and time to invest considerable quantities of time in developing their mathematical potential to the full.

7. My seventh belief has to do with the amount of time that students will commit to their study and their homework. In Australian schools, students spend about thirty hours per week at school. Yet, if they're fifteen or sixteen or seventeen years of age and left school, they'd probably spend thirty five or forty hours per week working — and some of them longer with overtime. **It seems quite reasonable to me to expect students to spend at least thirty five or forty hours a week, in their final years of school, learning.** And that means a minimum of five to ten hours per week doing homework and study. That doesn't seem at all unreasonable to me. Now, students who want to excel will spend more time than that. A student who wants to become top of their class will study to a certain level. If they want to top their entire year at school, they'll study harder. If they want to become one of the best in their region, they must study harder still. And if they want to become good in their state or on a national level it requires yet more work. This is true in every endeavour and it does not matter about your talent. The people who run and swim in the Olympic Games are incredibly talented athletes, yet every single one of them works very, very hard to get there. I want to encourage you to think about your training academically in much the same way. As a personal confession I want to share that, when I was fifteen, I was studying thirty five hours per week at home on top of my thirty hours of school. When I was sixteen, the figure was forty hours. When I was seventeen, it was forty five hours plus. Now, I was particularly dedicated and my reward for doing that was that I performed quite well at the state level in every subject. If you wish to perform well at your class level or your school level or your state level, you choose the amount of time



**There's not much point going to tutoring or going to classes if you're not going to bother learning the material.**

*It seems quite reasonable to me to expect students to spend at least thirty five or forty hours a week, in their final years of school, learning.*



**Not everyone needs to become a mathematician.**



Devote time to developing your brain at the very time your brain is developing.

*Train yourself to think clearly and present your work clearly so that it not only reflects your good clear thinking but it also INFLUENCES your thinking for the better!*



It is ok to excel in mathematics!

you're prepared to commit, but don't disparage people who want to commit a considerable amount of time any more than you would disparage someone who trains very hard at their music or very hard at their sport.

8. My eighth belief is that **it's good for your brain's development to do some study**, and I'm actually quite concerned about students who don't. When I was at university, for a short while I studied neurophysiology (and quite enjoyed it) and I've tried to keep up some reading in the field over the years. I'm far from an expert and what I'm about to share is definitely not a proven thing — it's a belief shared by some in the field — that the brain is being refined during the teenage or adolescent years according to how it is being used. Jay Giedd from McGill University in Quebec, Canada (he and his research team have been studying teenage brain development among other things) [says](#) that "If a teen is doing music or sports or academics, those are the cells and connections that will be hardwired. If they're lying on the couch or playing video games or MTV, those are the cells and connections that are going to survive." I want you to consider just what you're investing in for your future and to encourage you to devote some time to developing your brain at the very time your brain is developing.

9. My ninth belief is that **you should follow your passion**. There are many, many things that I would do differently if I had my life over again and this is one of them. As I've already shared, when I was at school I spent many hours studying and, sad to say, a large proportion of that time was spent studying things that I really disliked intensely. Unfortunately, only about 5% (or less than 5%) of my study time was devoted to mathematics, my primary passion. Fortunately, I still managed to come in the top handful of students out of (I think at that time) about 30 thousand students. I've often wondered, if I'd devoted more time to my mathematics, whether I would have en-

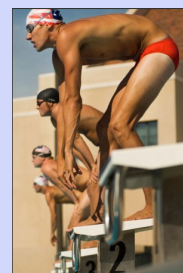
joyed my study vastly more than I did and performed (in mathematics) considerably better than I did. I encourage you to do the same. Make sure that you have the basics in place — that is, mathematics up to about the age of thirteen or fourteen in school — the rudiments, if you like, for living in our society — but, once that's in place, pursue your passion. If it's mathematics, devote time to that. If your passions lie elsewhere, then spend most of your time in those areas. Don't neglect your mathematics, but don't make them a major part of your studies if that's not your passion.

10. My tenth belief is that **it's vital that you set your work out clearly** when doing your mathematics, and this is something I'm quite passionate about. Now, I can't prove what I'm about to share, but I grew up at a time when a great deal of research was being done in a field called biofeedback (read about it on the [Wikipedia site](#) and at [Psychotherapy.com](#)). I have a theory that what you put on your paper reflects what is going on in your mind, and also it feeds back into your mind and influences how you think. There is a loop or a circuit being completed. If you're thinking in a messy way, and put your work out in a messy form on paper, then what you read and what goes back into your mind is also messy and confusing. I believe firmly in training yourself to think clearly and to present your work clearly so that, as you proceed through a problem, and set your work out in a clear and stepwise fashion, that it not only is reflecting your good clear thinking but it's also INFLUENCING your thinking for the better! I therefore encourage you to set your work out clearly! You'll notice that I emphasis clear setting out in the mathematics videos that I produce — so please watch out for it.

My purpose in sharing these these ten beliefs with you was to help you examine and clarify your own attitudes, beliefs and approaches to studying mathematics.

I hope that these principles can help you have a more balanced and enjoyable lifestyle, a more workable study system and greater success in your examinations!

## Notes



**This is your chance to plan a really good lifestyle change!**

**Why not 'dive in' and start today?**

*Use this page to make notes about the most important things you have learned from this booklet.*

*Make a list of things you plan to put into practice, sort them in order of priority and start on the first item today!*



**As you make lifestyle changes, make sure you obtain the support of those around you.**

**With the help of adults and peers you will find it so much easier to make the changes you want.**

VISIT OUR  
WEBSITE FOR  
**FREE HELP**  
WITH YOUR  
MATHEMATICS!

[www.crystalclearmaths.com](http://www.crystalclearmaths.com)

You are learning and studying all your life. Studying is not difficult. We naturally learn things that interest us, provide us with variety and a creative outlet, and give us opportunity to practice them with like-minded people.

Many students find mathematics difficult because they find it to be a boring subject, lacking variety and creativity, and they feel that no one around them likes it either.

The solution is 'simple.' Seek out like-minded and stimulating people and resources, search for variety and challenges that may not exist in your textbook, and challenge yourself to find things of interest within the subject.

Look inside for lots of advice and suggestions to change your mathematical future!

## About Crystal Clear Mathematics

Crystal Clear Mathematics is a private venture by Graeme Henderson, a semi-retired mathematics teacher in Australia.

With the support of his family and a few former students he started planning the website in June 2012. The website, YouTube channel and FaceBook page were launched during October 2012.

School classrooms can be an exceedingly difficult place to learn mathematics well. Teachers vary in passion and ability (as is natural), class behaviour and attitudes (and bullying) can alter the learning environment radically, learning in a large group at the same speed and with little opportunity to receive one-on-one help leads to boredom or frustration for many/most students, textbooks vary in quality and lack a lot of stimulating material (publishers restrict the size of the books), and sometimes parents and peers share their own bad experiences or attitudes with a student so that he/she adopts the same attitudes.

Graeme has decided to share his insights and resources via video (YouTube) and downloadable resources so that students can learn topic-by-topic in a stimulating way that makes sense.

This is a long-term project, so please be patient and feel free to make suggestions via the website.

### Copyright

Although he holds copyright on all the material that he produces (including this 'How to Study' manual), everything that he produces is available for **FREE DOWNLOAD** worldwide as long as no commercial gain is made from it. He is determined that people from poorer families and from developing nations have free access to quality training materials in mathematics.

If you can afford to support this ministry, any donation that you can make via the website will be gratefully received.

Thank you.



**Graeme Henderson**  
**B.Sc. (Hons), B.Th. (Hons),**  
**Dip.Ed., Dip.Min.**