Learning the Quadratic Formula

by

Graeme Henderson
Crystal Clear Mathematics (2013)

Derivation of the Quadratic Formula

The general quadratic equation $y = ax^2 + bx + c$ describes a parabola.

To find the values of x (roots or zeros) where the parabola crosses the x-axis, we solve the quadratic equation simultaneously with the equation for the x-axis, $y = 0$. This means that we are seeking solutions to the quadratic equation $ax^2 + bx + c = 0$.

The quadratic formula is derived from this equation and finds its solutions.

The derivation is as follows:

$$ax^2 + bx + c = 0$$

Divide by 'a' to make the quadratic a monic one, i.e. so that the coefficient of x^2 is 1.

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Subtract the constant term c/a from both sides.

$$x^2 + \frac{bx}{a} = -\frac{c}{a}$$

Complete the square. Remember to halve the coefficient of x (half of b/a is b/2a) and square the result. This constant should be added to both sides of the equation

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Write the LHS as a perfect square and expand the square on the RHS.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

Combine the fractions on the RHS by multiplying the top and bottom of c/a by 4a.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Take positive square root of both sides.

$$\left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2 - 4ac}}{2a}$$

Remove the absolute value sign.

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

Subtract the constant term b/2a from both sides.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

… and this is the finished quadratic formula!

You may follow this derivation by watching my YouTube video.

This Excel Workbook contains randomised worksheets designed to help you practise your use of the quadratic formula (and much more).

crystal clear
MATHEMATICS

How to Learn This Formula

My personal preference is to learn each formula in the context of mathematics that I already know and understand. The simplest method to achieve this is by deriving the formula from an equation that I already know using mathematical skills that are already familiar to me.

In this way, I not only learn the formula, I fully understand its context, I rehearse some earlier mathematics that is important for the understanding of this new mathematics, and I practice using some important mathematical skills at the same time!

In this case, I am practicing the very important skill of 'completing the square.'

Here are the steps to follow:

- Set time aside (without distractions) to learn and practice this deriveation. I.e. focus intently on learning this! You will learn much more rapidly and efficiently.

- Practise deriving the formula at least once per day until you are satisfied that the entire process is now 'obvious' to you

- Practice using this formula to solve quadratic equations or difficult problems. Do as many of these as you can manage during the first few days that you are learning about this area of mathematics. The secret is to do it sooner rather than later, more rather than less, and try to solve a great variety of questions as well. This is where a good text book can make a huge difference in how well you learn. Variety is important! Download my Excel Workbook for useful drill exercises.

- Each day during this intensive learning time, ask yourself two questions: "Do I understand this?" and "Am I fast enough?" Continue practising the derivation and use of the formula until you can honestly answer, "Yes!" to both questions.

MASSED PRACTICE

- When you are confident that you understand the material and can work quickly, ask yourself a final question: "How long before I am likely to forget this?" Place a reminder in your diary or calendar to practise the derivation and solve a few more problems before you have a chance to forget this work.

- Repeat this last step as often as necessary.

DISTRIBUTED PRACTICE

I hope you find this useful. I will be producing further booklets explaining how to use the quadratic formula, how to factorise quadratic expressions, how to solve quadratic equations, and how to graph parabolas. Please check my website (see footer below) regularly to see if I have added more material.

See my How to Study booklet to learn how to set up an effective study system, how to study your mathematics, and much more!

crystal clear
MATHEMATICS

Other Ways to Learn the Formula

There are other ways to memorise this formula as well.

You might find these YouTube links useful:

Using a melody: [Row, Row, Row Your Boat](); [Jingle Bells](); [Mary Had a Little Lamb](); [Pop Goes the Weasel](); [Pop Goes the Weasel]() (with a cute child, showing that anyone can learn the lyrics); and the [Quadratic Formula Song]() by Michael Kelly.

Using rhythm (rap): I couldn't find many with good sound quality but [here]() is an example.

Using a story: [This one]() is quite clever and short.

Using the Memory Palace and other skills: Watch this [TED Talk]() by Joshua Foer to learn a bit about clever memory techniques and embark on a new adventure!

Be inspired to experiment with memory techniques that may work for you!

Hoping you enjoy your mathematics!

Graeme Henderson (June 2013)

_____